COMPI215 - FOUNDATIONS OF COMPUTER SCIENCE

Lectures 1 & 2 - Introduction & Sets
Pawel Sobocinski
ADMINISTRATIVE ISSUES
YOUR LECTURERS

• Dr. Pawel Sobocinski (module leader, weeks 1-4)
• Dr. Srindandan Dasmahapatra (weeks 5-8)
• Dr. Kate Farrahi (weeks 9-10)

• Then revision and exams
PEER INSTRUCTION

• My lecturing style
  • lecture for up to 15 minutes, then ask you a question
  • you think about it for 30 seconds and discuss it with the person next to you for a couple of minutes
  • then we figure out the answer together

• Slides, exercise sheets and other resources available at the COMP1215 homepage https://secure.ecs.soton.ac.uk/notes/
ADVICE

• Not all modules are created equal
• do not fall behind
  • come to lectures
  • come to tutorials
  • come to additional sessions
• if you miss a session, make an appointment with somebody who didn’t
  and catch up!

DON’T PANIC

• there will be additional help available
TIMETABLE

• **Lectures**
  
  • Mondays 14:00-16:00 46/3001
  
  • Fridays 10:00-11:00 46/3001
  
  • additional slot, Fridays 14:00-15:00

• **Tutorials** (interactive problem solving sessions)
  
  • **starting next week**
    
    • Mondays 17:00-18:00 35/1001
    
    • Tuesdays 17:00-18:00 35/1001
ASSESSMENT

• 5 tests, together worth 25% (5% each)
  • likely held on Fridays in weeks 3, 5, 7, 9, 11

• 2 hr exam worth 75%
  • solving tutorial questions and revision will prepare you
  • past exams are available
LECTURES

• slides available a few days before each lecture
• read the slides and the recommended reading before coming to class
• lectures will not necessarily follow the slides, they will be used to clarify concepts
QUESTIONS?
INTRODUCTION
TOPICS COVERED

• Much of computer science relies on **discrete mathematics**
• The major topics that we will study are:
  • **Set theory**
    • (used in formal methods, theoretical computer science, algorithms, data structures, …)
  • **Propositional and predicate logic**
    • (used in programming languages, artificial intelligence, cybersecurity, formal methods, …)
  • **Combinatorics**
    • (used in problem solving, algorithm design, complexity theory, …)
  • **Linear algebra**
    • (used in artificial intelligence, machine learning, quantum computing, cyber physical systems, …)
  • **Probability and statistics**
    • (used in data science, research methods, multi-agent systems, …)
WHAT IS DISCRETE MATHS?

• Discrete vs Continuous
  • roughly speaking
    • you can **count** discrete things
      • balls, people, planets, seconds, ... 
    • you can **deform** (e.g. stretch and shrink) continuous things
      • rubber bands, routes, curves, space, time, ... 

• continuous mathematics is important in the physical sciences, and includes differential calculus, topology, ... 
  • real numbers, complex numbers 

• discrete mathematics is important in the logical sciences: computers handle discrete data (bits)
  • data structures (trees, graphs, stacks, multisets, sets, ...), programs 
  • the set of natural numbers \( \mathbb{N} = \{0, 1, 2, ... \} \)
WHY DO COMPUTER SCIENTISTS NEED DISCRETE MATHS?

High level skills you will learn in this module

• **importance of good abstractions**
  • difference between spaghetti code and well-structured code

• **importance of being rigorous**
  • difference between buggy code and robust code
IMPORTANCE OF GOOD ABSTRACTIONS

• Writing a computer program is the task of:
  • inventing an **abstraction**.
    • “database”, “operating system”, etc
  • these abstractions have become subjects!
• and **making it formal**
  • implementing it, writing the code
• A good programmer has a supply of reusable abstractions
  • **data structures**
  • design patterns
GREEK ALPHABET

• Because maths uses a lot of letters, sometimes we run out, and so traditionally greek is used as a backup
• Here are some letters you will see used fairly often:

  • α (alpha)
  • β (beta)
  • γ (gamma)
  • δ (delta)
  • ε (epsilon)
  • µ (mu)
  • σ (sigma)
  • φ (phi)
  • Ψ (psi)
  • ω (omega)
QUESTIONS?
SETS

BASIC DEFINITIONS, NOTATION AND EXAMPLES
SETS

• It is useful to imagine a set as a bag of elements

• The shape of the bag and position of elements are not important, the next picture represents the same set
• element repetition does not matter: the next two pictures represent the same set

• the only thing that matters is whether an element is in a set or not

• if sets $X$ and $Y$ contain the same elements then we say that $X=Y$
QUESTION

• How many different sets appear on this slide?
  a. 1
  b. 2
  c. 3
  d. 4
NOTATION

- Sets are usually specified with curly braces
  - \{0,1\} is the set with elements 0 and 1
  - order of elements is not important
    - \{0,1\} and \{1,0\} denote the same set with 2 elements
  - multiplicity of elements is not important
    - \{0,1,0\} and \{0,1\} denote the same set
- we sometimes use dots as shorthand
  - eg. \{0,...,9\} is the set of digits \{0,1,2,3,4,5,6,7,8,9\}

- dots are sometimes used as shorthand for infinitely many elements
  - \{0,1,2,...\} is the set of non negative integers
  - \{0,2,4,...\} is the set of even numbers
MEMBERSHIP

• When an element $x$ is in a set $X$ we write $x \in X$
• When an element $y$ is not in a set $X$ we write $y \not\in X$

• $a \in \{b, a, c\}$
• $0 \in \{0, 1\}$
• $i \not\in \{t, e, a, m\}$
CARDINALITY (OF FINITE SETS)

• For finite sets, write $|X|$ for the number of elements of $X$.

• this is the **cardinality** of $X$.

• $|\{0,1,2,3,4,5,6,7,8,9\}| = 10$

• $|\{a,b,a,c\}| = 3$

• cardinality can be extended to infinite sets, more on this later!
SOME OTHER SETS FROM MATHS

\[
\begin{align*}
\mathbb{N} & \overset{\text{def}}{=} \{0, 1, 2, \ldots\} & \text{the natural numbers} \\
\mathbb{Z} & \overset{\text{def}}{=} \{\ldots, -2, -1, 0, 1, 2, \ldots \} & \text{the integers} \\
\mathbb{R} & \text{the reals} \\
\mathbb{C} & \text{the complex numbers}
\end{align*}
\]

All these are examples of \textit{infinite} sets.
Are all infinities the same? We will answer this soon!
EMPTY SET

- **Recall**: A set is a collection of elements

- If a set does not contain any elements it is called the **empty set** and is usually denoted $\emptyset$
  
  - $|\emptyset| = 0$
Q. How many different empty sets are there?
OPERATIONS ON SETS
UNIONS OF SETS

$A \cup B$

$A \cup B$
INTERSECTIONS OF SETS

A \cap B

A \cap B
DIFFERENCE OF SETS

Q. What’s B - A?
QUESTION

• What’s wrong with the “definitions" on the previous slides?
A predicate $\phi(x)$ is something that is either true or false about a member $x$ of some universe. We say that it holds for $a$ or that $a$ satisfies $\phi$ when $\phi(a)$ is true.

Examples:

- $\phi(x) = \text{"x is called Michael"}$ is probably holds for some people in this room and doesn't hold for most
- $\phi(x) = \text{"x is an apple"}$ is a predicate in the universe of fruit
- $\phi(x) = \text{"x is even"}$ is a predicate about the natural numbers
  - 2 satisfies $\phi$, 741 does not

We will talk more formally about properties in due course (logic!)
NAIVE SET THEORY

• Given a predicate $\varphi$ we can construct a subset of $X$ that contains the elements that satisfy $\varphi$

• The subset of $X$ that contains those elements that satisfy $\varphi$ is usually denoted $\{ x \mid x \in X \& \varphi(x) \}$

  • eg. $\{ x \mid x \in \mathbb{N} \& x \text{ is even} \} = \{ 0, 2, 4, \ldots \}$

  • $\{ x \mid x \in \mathbb{R} \& x^2 + x - 6 = 0 \} = \{ 2.0, -3.0 \}$

Q. What is the difference between $X$ and $\{ x \mid x \in X \}$?
ELEMENTARY OPERATIONS ON SETS

• Union
  • \( X \cup Y = \{ z \mid z \in X \text{ or } z \in Y \} \)
  • associative ( \( X \cup (Y \cup Z) = (X \cup Y) \cup Z \) )
  • commutative ( \( X \cup Y = Y \cup X \) )

• Intersection
  • \( X \cap Y = \{ z \mid z \in X \text{ and } z \in Y \} \)
  • associative and commutative

• Difference
  • \( X - Y = \{ z \mid z \in X \text{ and } z \notin Y \} \)
  • \textbf{not} associative and \textbf{not} commutative (find \textit{counterexamples}!)
MATCH THE ENGLISH LANGUAGE TEXT TO THE STATEMENT OF SET THEORY

i) John and Kat do not have any friends in common
   a) $\text{friends}(\text{Al}) \cap \text{friends}(\text{Kat})$  
   b) $\text{friends}(\text{Al}) \subseteq \text{friends}(\text{John}) \cup \text{friends}(\text{Kat})$  
   c) $\text{friends}(\text{John}) \cap \text{friends}(\text{Kat}) = \emptyset$  
   d) $\text{friends}(\text{John}) - \text{friends}(\text{Al}) = \text{friends}(\text{Kat})$  

ii) all people that are friends with Al are also either friends with John or Kat

iii) common friends of Al and Kat

iv) Kat’s friends are those friends of John that are not friends with Al
REASONING ABOUT SETS
HOW TO RELATE SETS?

• A basic relationship between sets is the **subset** relation
  • A set $X$ is a subset of $Y$ when every element of $X$ is also an element of $Y$
    • notation: $X \subseteq Y$
  • **fact:** every set satisfies $X \subseteq X$.
  • **fact:** for any set $X$ we have $\emptyset \subseteq X$
  • it is sometimes useful to talk about sets that are **strictly** larger or smaller, we write $X \subset Y$ to mean that $X \subseteq Y$ and there exists $y \in Y$ and $y \notin X$. 


WHEN ARE TWO SETS EQUAL?

• A set is completely defined by its elements

• **Proof Technique:** To show that sets \( X \) and \( Y \) are equal it is enough to show
  • \( X \subseteq Y \) and
  • \( Y \subseteq X \)
QUESTION

• Suppose that $X, Y$ and $Z$ are sets. Is it always the case that $X \cup (Y \cap Z) = (X \cup Y) \cap (X \cup Z)$?
USING VENN DIAGRAMS

- Can be useful for intuitions (both for proof and for counterexamples) about small problems
- quickly become unusable when many sets are involved

- Eg. Look at \( Xu(Y \cap Z) = (XuY) \cap (XuZ) \)

- what about \( Xu(Y_1 \cap Y_2 \cap \ldots \cap Y_k) \)?
Now we prove
\[(X \cup Y) \cap (X \cup Z) \subseteq X \cup (Y \cap Z)\] (**)

\(x \in (X \cup Y) \cap (X \cup Z)\) so both of the following are true
\[x \in X \cup Y \quad (1) \quad \text{and} \quad x \in X \cup Z \quad (2)\]

Now either

**Case 1:** \(x \in X\)

if \(x \in X\) then \(x \in X \cup (Y \cap Z)\)

**Case 2:** \(x \notin X\)

if \(x \notin X\) then because \((1)\), we must have \(x \in Y\)

because \((2)\), we must have \(x \in Z\)

so \(x \in Y \cap Z\) which implies
\[x \in X \cup (Y \cap Z)\]

we made no assumptions about \(x\) so (***) must be true
QUESTION

- Prove that for all sets $X, Y$ and $Z$, we have $X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$