1. Let \( A \overset{\text{def}}{=} \{0, -1, 5\} \) and \( B \overset{\text{def}}{=} \{1, 3, 0, 3\} \). List all the elements of the following sets using the standard curly brace notation. For each set give its cardinality:

a) \( A \cup B \);
b) \( A \cap B \);
c) \( A \times B \);
d) \( A - B \);
e) \( (A \times \{0\}) \cup (B \times \{1\}) \);

2. Find a predicate \( \varphi \) to express the following sets of integers using the notation \( \{x \mid \varphi(x)\} \).

a) The set of all natural numbers divisible by 7;
b) \( \{x \mid x \geq 0 \text{ and } x < 1000\} \cap \{x \mid x > -10 \text{ and } x \leq 10\} \);
c) \( \{0, 1\} \times \{x \mid x \geq 0 \text{ and } x \leq 5\} \);

3. Prove the following. You can use Venn diagrams for inspiration.

a) \( X \cup (Y \cup Z) = (X \cup Y) \cup Z \);
b) \( X \cap (Y \cap Z) = (X \cap Y) \cap Z \);
c) \( X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z) \);
d) \( X \cup (Y \cap Z) = (X \cup Y) \cap (X \cup Z) \);
e) \( X \cup (Y_1 \cap \cdots \cap Y_k) = (X \cup Y_1) \cap \cdots \cap (X \cup Y_k) \) for any \( k > 0 \) and sets \( Y_1, \ldots, Y_k \).
4. Suppose $X, Y$ and $Z$ are sets. Does $X \times (Y + Z) = X \times Y + X \times Z$?
   If $X, Y$ and $Z$ are finite, what can we say about the cardinalities of
   $X \times (Y + Z)$ and $X \times Y + X \times Z$?

5. Suppose that $X$ and $Y$ are sets and $X \times Y$ is their cartesian product.
   There are two functions known as the projections: $\pi_1 : X \times Y \to X$ and
   $\pi_2 : X \times Y \to Y$; defined $\pi_1(x, y) = x$ and $\pi_2(x, y) = y$ respectively.
   
   a) show that given a third set $Z$ and functions $f : Z \to X$, $g : Z \to Y$,
      there exists a function $h : Z \to X \times Y$ satisfying $h; \pi_1 = f$ and
      $h; \pi_2 = g$; (hint: start by drawing a diagram with all the functions)
   b) show that the function $h$ is the unique such function; i.e. if there
      exists $h'$ with $h'; \pi_1 = f$ and $h'; \pi_2 = g$ then $h = h'$.

6. Suppose that $X$ and $Y$ are sets and $X + Y$ is their sum. There are two
   functions known as the injections: $i_1 : X \to X + Y$ and $i_2 : Y \to X + Y$;
   defined $i_1(x) = (x, 0)$ and $i_2(y) = (y, 1)$ respectively.
   
   a) show that given a third set $Z$ and functions $f : X \to Z$, $g : Y \to Z$,
      there exists a function $h : X + Y \to Z$ satisfying $i_1; h = f$ and
      $i_2; h = g$;
   b) show that $h$ is the unique such function.