COMP2210 CLASS TEST 3

Answers must be copied to the answer sheet or they will be ignored!
The test has 20 questions to complete in 45 minutes.
No documents allowed. The use of electronic calculators is forbidden.

Question 1 ♠ For the following set, choose the two options that apply.
\{ M \mid M \text{ is a Turing machine which accepts the empty string} \} (2 marks)

A The set is not recursively enumerable.
■ The set is not recursive.
C The set is recursive.
■ The set is recursively enumerable.

Question 2 Can the tape alphabet \( \Gamma \) of a Turing machine be the same as the input alphabet \( \Sigma \)? (1 mark)

A YES ■ NO

Question 3 ♠ Let \( M \) be a Turing machine which at each step in its computation can only write back the same symbol as the one it reads, and then move right. Which of the following statements are true: (2 marks)

A \( L(M) \) need not be recursive.
■ \( L(M) \) is recursive.
C \( L(M) \) need not be regular.
■ \( L(M) \) is regular.

Question 4 Is the set of recursive languages closed under intersection? (1 mark)

■ YES B NO

Question 5 Is the set of recursive languages closed under the star operation \( * \)? Where for a language \( L \subseteq \Sigma^* \), the language \( L^* \) is defined by

\[ L^* = \{ w_1 \ldots w_k \mid k \geq 0 \text{ and } w_i \in L \text{ for each } i \in \{1,\ldots,k\} \} \]

(2 marks)

■ YES B NO

Question 6 ♠ For the following set, choose the two options that apply.
\{ M \mid M \text{ is a DFA which accepts } w^R \text{ whenever it accepts } w \}. \text{ Here, for a word } w, w^R \text{ denotes the reverse of } w. (2 marks)

A The set is not recursive.
■ The set is recursive.
C The set is not recursively enumerable.
■ The set is recursively enumerable.
Corrected

**Question 7**  Can a Turing machine ever write the blank symbol $\Box$ on its tape?  
(1 mark)

[ ] YES  [ ] NO

**Question 8** ♣  For the following set, choose the two options that apply.  
\{ $A\#B$ | $A$ and $B$ are DFAs accepting the same language \}  (2 marks)

[ ] The set is recursively enumerable.
[ ] The set is not recursively enumerable.
[ ] The set is not recursive.
[ ] The set is recursive.

**Question 9**  Can a Turing machine contain just a single state?  (1 mark)

[ ] NO  [ ] YES

**Question 10**  Is the set of recursively-enumerable languages closed under union?  
(1 mark)

[ ] NO  [ ] YES

**Question 11**  Is the set of recursively-enumerable languages closed under complement?  (1 mark)

[ ] NO  [ ] YES

**Question 12** ♣  For the following set, choose the two options that apply.  
\{ $M\#x$ | $M$ is a Turing machine, $x$ is an input for $M$ and $M$ loops on $x$ \}  (2 marks)

[ ] The set is not recursively enumerable.
[ ] The set is recursive.
[ ] The set is not recursive.
[ ] The set is recursively enumerable.

**Question 13**  Is the set of recursively-enumerable languages closed under the star operation $^*$?  Where for a language $L \subseteq \Sigma^*$, the language $L^*$ is defined by

$$L^* = \{ w_1 \ldots w_k \mid k \geq 0 \text{ and } w_i \in L \text{ for each } i \in \{1, \ldots, k\} \}$$

(2 marks)

[ ] YES  [ ] NO
Question 14  Is the following a correct and complete definition of the transition function of a Turing machine over the input alphabet \( \{a\} \), with states \( \{s, t, r\} \), \( s \) as the initial state, \( t \) as the accept state and \( r \) as the reject state? Here, correctness refers to the absence of incorrect information, whereas completeness refers to no missing information. (1 mark)

\[
\begin{array}{c|ccc}
\vdash & a \\
\hline
s & (s, a, R) & (s, a, R) \\
\end{array}
\]

[ ] A complete but not correct  [ ] neither correct nor complete  [ ] B both correct and complete  [ ] D correct but not complete

Question 15  Is the set of recursively-enumerable languages closed under the concatenation operation? Where for two languages \( L_1 \) and \( L_2 \), their concatenation is given by

\[
L_1 \cdot L_2 = \{w_1w_2 \mid w_1 \in L_1, w_2 \in L_2\}
\]

(2 marks)

[ ] A NO  [ ] B YES

Consider a Turing machine over the input alphabet \( \{0, 1\} \), with set of states \( \{s_0, s_1, s_2, t, r\} \), initial state \( s_0 \), accept state \( t \), reject state \( r \), and with transition function given by:

\[
\begin{array}{c|cccc}
\vdash & 0 & 1 & \sqcup \\
\hline
s_0 & (s_0, \vdash, R) & (s_1, 0, L) & (s_1, 1, L) & (s_0, \sqcup, R) \\
s_1 & (s_1, \vdash, R) & (s_2, 1, R) & (s_1, 0, R) & (s_2, 1, R) \\
s_2 & (r, \vdash, R) & (s_2, 0, R) & (s_2, 1, R) & (t, \sqcup, L) \\
\end{array}
\]

Answer the following 5 questions.

Question 16  Is the machine \( M \) total? (1 mark)

[ ] A NO  [ ] B YES

Question 17  What is the outcome of executing \( M \) on input \( \vdash \)? (1 mark)

[ ] A \( M \) halts and accepts.  [ ] B \( M \) halts and rejects.  [ ] C \( M \) does not halt.

Question 18  What is the outcome of executing \( M \) on input \( \vdash 0 \)? (1 mark)

[ ] A \( M \) halts and accepts.  [ ] B \( M \) does not halt.  [ ] C \( M \) halts and rejects.
Question 19  What is the outcome of executing $M$ on input $\vdash 110$ ? (1 mark)

A  $M$ does not halt.
B  $M$ halts and accepts with $\vdash 000$ on the tape.
C  $M$ halts and rejects with $\vdash 011$ on the tape.
D  $M$ halts and accepts with $\vdash 001$ on the tape.
E  $M$ halts and accepts with $\vdash 111$ on the tape.

Question 20  How many times does the machine reach a configuration where the head points to the left endmarker (including the initial configuration), when started on the input $\vdash 111110$ ? (2 marks)

A  3 times.
B  2 times.
C  4 times.
D  1 time.
Enter your registration id on the left boxes and write your first name and last name below.

Firstname and lastname:

............... 

Answers must be given exclusively on this sheet: answers given on the other sheets will be ignored. Please fill completely with your pen the box of the answer you want to select (ticking it or crossing it is not enough).

Question 1: A □ C □
Question 2: A □
Question 3: A □ C □
Question 4: □ B
Question 5: □ B
Question 6: A □ C □
Question 7: □ B
Question 8: □ B □ C □
Question 9: □ B
Question 10: A □
Question 11: □ B
Question 12: □ B □ D
Question 13: □ B
Question 14: A □ B □ D
Question 15: A □
Question 16: □ B
Question 17: A □ B □
Question 18: □ B □ C
Question 19: A □ B □ C □ E
Question 20: A □ C □ D