COMP2210 CLASS TEST 4

Answers must be copied to the answer sheet or they will be ignored!
The test has 18 questions to complete in 45 minutes.
No documents allowed. The use of electronic calculators is forbidden.

Question 1 ♣ Suppose a non-deterministic Turing machine solves the Hamiltonian path problem (HAMPATH) in \(O(n)\) time. Based on this information, which of the following statements are true? (2 marks)

A There exists a deterministic Turing machine which solves HAMPATH in \(2^{O(n)}\) time.
B There exists a deterministic Turing machine which solves HAMPATH in \(O(n^2)\) time.
C There exists a deterministic Turing machine which solves HAMPATH in \(O(2^n)\) time.

Question 2 ♣ A path in a directed graph is called simple if it does not repeat any nodes. Let LPATH be the problem of deciding whether a given graph contains a simple path of length at least \(k\) between two given nodes. Which of the following statements are true? (3 marks)

A LPATH is in P.
B LPATH is NP-complete.
C LPATH is in NP.

Question 3 Is the following true or false? \(2^n\) is \(\Theta(3^n)\) (1 mark)

A False
B True

Question 4 Let X be an NP-hard problem, and suppose we find a polynomial-time algorithm for X. Does this mean that we can solve SAT (the boolean satisfiability problem) in polynomial time? (1 mark)

A Yes
B No

Question 5 ♣ Let \(L\) be a language such that a reduction exists from \(L\) to the Halting set. Which of the following statements are true? (2 marks)

A \(L\) may or may not be recursively enumerable.
B \(L\) is recursively enumerable.
C \(L\) may or may not be recursive.
D \(L\) is not recursive.
Question 6  Let CONNECTED be the problem of deciding whether a given directed graph is connected. A graph is connected if any node can be reached from any other node by travelling along the edges of the graph. Is the statement ”CONNECTED is in P” true, false, or not known to be either true or false? (2 marks)

■ True  ■ B False  ■ C Not known

Question 7  Suppose a multi-tape deterministic Turing machine solves the Hamiltonian path problem (HAMPATH) in $O(n^2)$ time. Based on this information, which of the following statements are true? (2 marks)

■ There exists a non-deterministic Turing machine which solves HAMPATH in polynomial time.
■ B There exists a single-tape deterministic Turing machine which solves HAMPATH in $O(n^2)$ time.
■ There exists a single-tape deterministic Turing machine which solves HAMPATH in $O(n^6)$ time.

Question 8  Let X be a decision problem for which a solution is known to require at least $2n\log(n)$ steps. Let A be an algorithm which solves X with time complexity $\Theta(n\log(n))$. What is the strongest conclusion you can draw about the complexity of X? (1 mark)

A X is in P.  ■ B X is $O(n\log(n))$  ■ C X is $\Theta(n\log(n))$

Question 9  Is the following true or false? If two languages $L_1 \subseteq \Sigma^*$ and $L_2 \subseteq \Sigma^*$ are in P, then so is $L_1 \cdot L_2 = \{w_1w_2 \mid w_1 \in L_1, w_2 \in L_2\}$. (1 mark)

■ True  ■ B False

Question 10  Is the following true or false? If a function is $2^{O(n)}$, then it is $O(2^n)$. (1 mark)

A True  ■ B False

Question 11  Let $M$ be a DFA. Which of the following statements is true: (2 marks)

A The set $\{M' \mid M' \text{ is a Turing machine and } L(M') = L(M)\}$ is decidable for some choices of $M$ and undecidable for others.
■ B The set $\{M' \mid M' \text{ is a Turing machine and } L(M') = L(M)\}$ is always undecidable.
■ C The set $\{M' \mid M' \text{ is a Turing machine and } L(M') = L(M)\}$ is always decidable.
**Corrected**

**Question 12** Let PATH be the problem of deciding if a given directed graph admits a path between two given nodes. Is the following statement true or false? If PATH is not NP-complete, then $P \neq NP$. (2 marks)

- [ ] True
- [ ] False

**Question 13** Suppose we find an $O(n^2)$ algorithm for SAT (the boolean satisfiability problem). Does this mean that every problem in NP can be solved in time $O(n^2)$? (1 mark)

- [ ] Yes
- [ ] No

**Question 14** Is the following true, false, or not known to be either true or false? If a language $L \subseteq \Sigma^*$ is in NP, then so is $L^* = \{w_1 \ldots w_n \mid w_i \in L \text{ for } i = 1, \ldots, n\}$. (1 mark)

- [ ] True
- [ ] Not known
- [ ] False

**Question 15 ♣** Let $L$ be a language such that a reduction from the set

$$\{M\#x \mid M \text{ loops on } x\}$$

to $L$ exists. Which of the following statements are true: (2 marks)

- [ ] $L$ is not recursively enumerable.
- [ ] $L$ may or may not be recursively enumerable.
- [ ] $L$ may or may not be recursive.
- [ ] $L$ is not recursive.

**Question 16** Is the following true or false? If a language $L \subseteq \Sigma^*$ is in P, then so is its complement. (1 mark)

- [ ] True
- [ ] False

**Question 17** Is the following true or false? $n \log n$ is $O(n^2)$ (1 mark)

- [ ] False
- [ ] True

**Question 18** Is the following true or false? $n^2$ is $O(n \log^2 n)$ (1 mark)

- [ ] False
- [ ] True
Corrected

ANSWER SHEET

Enter your registration id on the left boxes and write your first name and last name below.

Firstname and lastname:

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Answers must be given exclusively on this sheet: answers given on the other sheets will be ignored. Please fill completely with your pen the box of the answer you want to select (ticking it or crossing it is not enough).

Question 1: □ B C
Question 2: A □ □
Question 3: □ B
Question 4: □ B
Question 5: A □ □ □ D
Question 6: □ B C
Question 7: □ B □
Question 8: A B □
Question 9: □ B
Question 10: A □
Question 11: A □ □ C
Question 12: □ B
Question 13: A □
Question 14: □ B C
Question 15: □ B C □
Question 16: □ B
Question 17: A □
Question 18: □ B