COMP2210 CLASS TEST 3

Answers must be copied to the answer sheet or they will be ignored!

The test has 15 questions to complete in 40 minutes.

No documents allowed. The use of electronic calculators is forbidden.

Question 1 In the proof of the undecidability of the halting problem we assume that a total Turing machine $K$ exists that decides the halting problem. We then use the diagonalisation proof technique to construct a new Turing machine $N$ that is not in the table. Which of the following is the correct construction of $N$?

A On the input $M_l\#x_l$ the Turing machine $N$:
   1. Runs $K$ on $M_l\#x_l$.
   2. If $K$ accepts then $N$ goes it to a trivial loop. If $K$ rejects then $N$ accepts.

B On the input $x_l$ the Turing machine $N$:
   1. Constructs the Turing machine $M_l$.
   2. Runs $K$ on $M_l\#x_l$.
   3. If $K$ accepts then $N$ accepts. If $K$ rejects then $N$ rejects.

C On the input $M_l\#x_l$ the Turing machine $N$:
   1. Saves the input $M_l\#x_l$.
   2. Runs $K$ on $\epsilon$.
   3. If $K$ accepts then $N$ goes it to a trivial loop. If $K$ rejects then $N$ accepts.

D On the input $x_l$ the Turing machine $N$:
   1. Constructs the Turing machine $M_l$.
   2. Runs $K$ on $M_l\#x_l$.
   3. If $K$ accepts then $N$ goes it to a trivial loop. If $K$ rejects then $N$ accepts.

E On the input $x_l$ the Turing machine $N$:
   1. Saves the input $x_l$.
   2. Runs $K$ on $\epsilon$.
   3. If $K$ accepts then $N$ goes it to a trivial loop. If $K$ rejects then $N$ accepts.

(9 mark, -5 mark)
Question 2  If $P$ is a semidecidable, but not decidable, property of strings in the alphabet $\Sigma$ then:

A  $\{x \mid P(x)\}$ is not recursive and $\{x \mid \neg P(x)\}$ is recursive.
B  $\{x \mid P(x)\}$ is recursive and $\{x \mid \neg P(x)\}$ is recursive.
C  $\{x \mid P(x)\}$ is recursive and $\{x \mid \neg P(x)\}$ is not recursive.
D  $\{x \mid P(x)\}$ is not recursive and $\{x \mid \neg P(x)\}$ is not recursive.

(3 mark, -1 mark)

Consider a Turing machine over the input alphabet $\{0, 1\}$, with the set of states $\{s_0, s_1, s_2, s_3, t, r\}$, initial state $s_0$, accept state $t$, reject state $r$, and with transition function given by:

<table>
<thead>
<tr>
<th>State</th>
<th>$\vdash$</th>
<th>0</th>
<th>1</th>
<th>$\sqcup$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_0$</td>
<td>$(s_0, \vdash, R)$</td>
<td>$(s_1, 1, R)$</td>
<td>$(s_1, 0, L)$</td>
<td>$(s_3, \sqcup, R)$</td>
</tr>
<tr>
<td>$s_1$</td>
<td>$(s_1, \vdash, R)$</td>
<td>$(r, 1, L)$</td>
<td>$(s_2, 0, R)$</td>
<td>$(s_1, \sqcup, R)$</td>
</tr>
<tr>
<td>$s_2$</td>
<td>$(t, \vdash, R)$</td>
<td>$(s_1, 0, L)$</td>
<td>$(s_1, 1, R)$</td>
<td>$(t, \sqcup, R)$</td>
</tr>
<tr>
<td>$s_3$</td>
<td>$(s_3, \vdash, R)$</td>
<td>$(t, 1, L)$</td>
<td>$(s_2, 1, R)$</td>
<td>$(s_0, 1, R)$</td>
</tr>
</tbody>
</table>

Question 3  What is the outcome of executing $M$ on input $\vdash$?

A  $M$ loops writing $\vdash \sqcup 1 \sqcup 1 \ldots$ (alternating sequence of blanks and 1s) on the tape.
B  $M$ loops writing $\vdash \sqcup 111 \ldots$ (infinite sequence of 1s) on the tape.
C  $M$ halts with $\vdash 11$ on the tape.
D  $M$ loops writing $\vdash 111 \ldots$ (infinite sequence of 1s) on the tape.

(3 mark, -1 mark)

Question 4  What is the outcome of executing $M$ on input $\vdash 11$?

A  $M$ loops writing $\vdash 111 \ldots$ (infinite sequence of 1s) on the tape.
B  $M$ rejects with $\vdash 11$ on the tape.
C  $M$ accepts with $\vdash 00$ on the tape.
D  $M$ loops writing $\vdash 1 \sqcup 1 \sqcup 1 \ldots$ (alternating sequence of 1s and blanks) on the tape.

(3 mark, -1 mark)
Corrected

**Question 5**  What is the outcome of executing $M$ on input $\vdash 0101$?

- $M$ loops with $\vdash 111\ldots$ (infinite sequence of 1s) on the tape.
- $M$ rejects with $\vdash 1101$ on the tape.
- $M$ rejects with $\vdash 101010$ on the tape.
- $M$ accepts with $\vdash 0110$ on the tape.

(3 mark, -1 mark)

**Question 6**  What is the outcome of executing $M$ on input $\vdash 1w$ where $w \in \{0, 1\}^*$?

- $M$ always accepts with $\vdash 1w$ on the tape and the head at position 2.
- $M$ always rejects with $\vdash 1w$ on the tape and the head at position 1.
- $M$ always rejects with $\vdash 1w$ on the tape and the head at position 2.
- $M$ always rejects with $\vdash 101010$ on the tape and the head at position 1.

(6 mark, -2 mark)

Given the Turing machine $M = (Q, \Sigma, \Gamma, \vdash, \sqcup, \delta, s, t, r)$, answer the following 3 questions.

**Question 7**  Only one of the following statements is true. Which one?

- $\Sigma \subseteq \Gamma$, $\vdash \in \Gamma$, and $\sqcup \in \Sigma$.
- $\Gamma \subseteq \Sigma$, $\vdash \notin \Sigma$, and $\sqcup \in \Gamma$.
- $\Sigma \subseteq \Gamma$, $\vdash \notin \Sigma$, and $\sqcup \in \Gamma$.

(2 mark, -1 mark)

**Question 8**  Which of the following is the correct signature for the transition function?

- $\delta : (Q - \{t, r\}) \times \Gamma \rightarrow Q \times \{L, R\}$
- $\delta : (Q - \{t, r\}) \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$
- $\delta : Q \times \Gamma \rightarrow Q \times \{L, R\}$
- $\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$

(2 mark, -1 mark)

**Question 9**  Only one of the following statements is true. Which one?

- $Q$ is finite and $t \neq r$.
- $Q$ can be infinite and $t$ can be the same as $r$.
- $Q$ can be infinite and $t \neq r$.
- $Q$ is finite and $t$ can be the same as $r$.

(2 mark, -1 mark)
Corrected

**Question 10**  If $P$ is a decidable property of strings in the alphabet $\Sigma$ then:

- A $\{x \mid P(x)\}$ is not recursive and $\{x \mid \neg P(x)\}$ is not recursive.
- B $\{x \mid P(x)\}$ is recursive and $\{x \mid \neg P(x)\}$ is recursive.
- C $\{x \mid P(x)\}$ is not recursive and $\{x \mid \neg P(x)\}$ is recursive.
- D $\{x \mid P(x)\}$ is recursive and $\{x \mid \neg P(x)\}$ is not recursive.

(3 mark, -1 mark)

**Question 11**  The set of recursive languages is

- A closed under intersection and complement but not union.
- B closed under union, intersection, and complement.
- C closed under union and intersection but not complement.
- D closed under union and complement but not intersection.

(3 mark, -1 mark)

**Question 12**  A configuration of a Turing machine is a pair $(s, w)$, where $s$ is a state, and $w$ is the contents of the tape.

- False.  
- B True.

(2 mark, -2 mark)

**Question 13**  A multi-tape Turing machine is more powerful than a single tape Turing machine.

- False.  
- B True.

(2 mark, -2 mark)

**Question 14**  The set of recursively enumerable languages is

- A closed under union and complement.
- B closed under union, intersection, and complement.
- C closed under union and intersection.
- D closed under intersection and complement.

(3 mark, -1 mark)

**Question 15**  A Turing machine can write the left end marker symbol $\vdash$ to the tape.

- True.  
- B False.

(2 mark, -2 mark)
Corrected

ANSWER SHEET

Fill your registration id on the left circles and write your first name and last name below.

Firstname and lastname:

.........................

Answers must be given exclusively on this sheet: answers given on the other sheets will be ignored. Please fill completely with your pen the box of the answer you want to select (ticking it or crossing it is not enough).

Question 1:  A  B  C  E
Question 2:  A  B  C
Question 3:  B  C  D
Question 4:  A  C  D
Question 5:  A  C  D
Question 6:  A  C  D
Question 7:  A  B  C
Question 8:  A  C  D
Question 9:  B  C  D
Question 10:  A  C  D
Question 11:  A  C  D
Question 12:  B
Question 13:  B
Question 14:  A  B  D
Question 15:  B