COMP2210 CLASS TEST 3

Answers must be copied to the answer sheet or they will be ignored!
The test has 17 questions to complete in 40 minutes.
No documents allowed. The use of electronic calculators is forbidden.

Question 1  (3pts, 0pt) Consider the following statements:

(i) The set of recursively enumerable languages is closed under unions.
(ii) The set of recursively enumerable languages is closed under intersections.
(iii) The set of recursively enumerable languages is closed under complements.

Which of the following statements are true:

A Only (iii) is true.
B (i), (ii) and (iii) are true.
C Only (ii) is true.
D Only (ii) and (iii) are true.
F Only (i) is true.
G Only (i) and (iii) are true.
H Neither (i) nor (ii) nor (iii) is true.

Question 2  (3pts, 0pt) Consider the following statements:

(i) The set of recursive languages is closed under unions.
(ii) The set of recursive languages is closed under intersections.
(iii) The set of recursive languages is closed under complements.

Which of the following statements are true:

B (i), (ii) and (iii) are true.
F Only (i) is true.
G Only (i) and (iii) are true.
H Only (ii) and (iii) are true.

Question 3  (3pts, -1pt) Consider the following statements:

(i) The set \( \{ M \# x \mid M \text{ halts on } x \} \) is recursively enumerable.
(ii) The set \( \{ M \# x \mid M \text{ loops on } x \} \) is recursively enumerable.

Which one of the following statements is true:

A Both (i) and (ii) are true.
B None of (i) and (ii) are true.
C Only (ii) is true.
D Only (i) is true.
Question 4  (4pts, -1pt) Consider the set: \( S = \{ A \mid A \text{ is a DFA accepting all strings over its input language} \} \). Which of the following statements are true:

A. \( S \) is recursive but not recursively enumerable.
B. \( S \) is both recursively enumerable and recursive.
C. \( S \) is recursively enumerable but not recursive.
D. \( S \) is neither recursively enumerable nor recursive.

Question 5  (2pts, -1pt) A Turing machine with two tapes can simulate a Turing machine with three tapes.

- True.
- False.

Question 6  (3pts, -1pt) Consider the set \( S = \{ M\#x \mid x \in L(M) \} \). Which one of the following statements is true:

A. \( S \) is both recursively enumerable and recursive.
B. \( S \) is neither recursively enumerable nor recursive.
C. \( S \) is recursive but not recursively enumerable.
D. \( S \) is recursively enumerable but not recursive.

Question 7  (2pts, -1pt) Can the tape of a Turing machine contain an infinite number of non-blank symbols at any point in the machine’s execution?

- No
- Yes

Question 8  (2pts, -1pt) Can the tape alphabet \( \Gamma \) of a Turing machine be the same as the input alphabet \( \Sigma \)?

- Yes
- No

Question 9  (5pts, -1pt) Given two languages \( L_1 \) and \( L_2 \), their concatenation is given by

\[ L_1 \cdot L_2 = \{ w_1w_2 \mid w_1 \in L_1, w_2 \in L_2 \} \]

Consider the following statements:

(i) If \( L_1 \) is recursive and \( L_2 \) is recursively enumerable, then \( L_1 \cdot L_2 \) is recursively enumerable.

(ii) If \( L_1 \) is recursively enumerable and \( L_2 \) is recursively enumerable, then \( L_1 \cdot L_2 \) is recursively enumerable.

Which of the following statements are true:

A. Only (i) is true.
B. Only (i) is true.
C. Both (i) and (ii) are true.
D. Neither (i) nor (ii) is true.
Question 10  (5pts, -1pt) Let M be a Turing machine which, on any input, computes as follows:

Step 1. scans to the end of the input;
Step 2. at each subsequent step in its computation, it writes back the same symbol as the one it reads and moves left (with or without changing state); if it reaches the left endmarker, it rejects.

Which one of the following statements is true?

A  \( L(M) \) is not always context-free, but is always recursive.
B  \( L(M) \) is not always regular, but is always context-free.
\( \square \) \( L(M) \) is always regular.
D  \( L(M) \) is not always recursive.

Consider a Turing machine \( M \) over the input alphabet \( \{0, 1\} \), with set of states \( \{s_0, s_1, s_2, t, r\} \), initial state \( s_0 \), accept state \( t \), reject state \( r \), and with transition function given by:

<table>
<thead>
<tr>
<th>( \sqsubset )</th>
<th>0</th>
<th>1</th>
<th>( \sqcup )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s_0 )</td>
<td>( (s_0, \sqsubset, R) )</td>
<td>( (s_1, 0, R) )</td>
<td>( (s_1, 1, R) )</td>
</tr>
<tr>
<td>( s_1 )</td>
<td>( (r, \sqsubset, R) )</td>
<td>( (s_1, 0, R) )</td>
<td>( (s_1, 1, R) )</td>
</tr>
<tr>
<td>( s_2 )</td>
<td>( (r, \sqsubset, R) )</td>
<td>( (s_2, 1, L) )</td>
<td>( (t, 0, R) )</td>
</tr>
</tbody>
</table>

Answer the following 5 questions.

Question 11  (2pts, -1pt) What is the outcome of executing \( M \) on an empty string?

A  \( M \) halps with a non-empty tape.
C  \( M \) halts with an empty tape.
\( \square \) \( M \) loops.

Question 12  (2pts, 0pt) What is the outcome of executing \( M \) on input 0?

A  \( M \) loops.
C  None of the other options.
D  \( M \) accepts with 1 on the tape.
E  \( M \) accepts with 10 on the tape.

Question 13  (3pts, 0pt) What is the outcome of executing \( M \) on input 1100?

A  None of the other options.
B  \( M \) loops.
D  \( M \) rejects.
E  \( M \) accepts with \( \sqsubset \) 1010 on the tape.
F  \( M \) accepts with \( \sqsubset \) 1101 on the tape.
Corrected

**Question 14** (4pts, -1pt) What is the outcome of executing $M$ on a non-empty string containing both 0s and 1s?

A. None of the other options.
B. $M$ always rejects strings of this form.
C. $M$ accepts some strings of this form and rejects others, but never loops.
D. $M$ always accepts strings of this form.

**Question 15** (3pts, -1pt) How many times does the machine reach a configuration where the tape head points to the left endmarker (including the initial configuration), when started on the input 000000?

A. None of the other options.
B. 2 times.
C. 3 time.
D. 1 time.

**Question 16** (2pts, -1pt) Can a Turing machine write the left endmarker symbol ⊢ anywhere on its tape?

A. No
B. Yes

**Question 17** (2pts, -1pt) Can a Turing machine have only one state?

A. No
B. Yes
Fill your registration id on the left circles and write your first name and last name below.

Firstname and lastname:

........................................

Answers must be given exclusively on this sheet: answers given on the other sheets will be ignored. Please fill completely with your pen the box of the answer you want to select (ticking it or crossing it is not enough).

Question 1: [A B C D E F]
Question 2: [B C D E F]
Question 3: [A B C ]
Question 4: [A B C D]
Question 5: [B]
Question 6: [A B C ]
Question 7: [B]
Question 8: [A]
Question 9: [A B E D]
Question 10: [A B E D]
Question 11: [A C]
Question 12: [A C D E]
Question 13: [A B E D F]
Question 14: [A B C]
Question 15: [A C D]
Question 16: [A]
Question 17: [B]