COMP2210 CLASS TEST 4

Answers must be copied to the answer sheet or they will be ignored!
The test has 17 questions to complete in 40 minutes.
No documents allowed. The use of electronic calculators is forbidden.

**Question 1** (2pts, -1pt) Consider the following statements:

(a) A language is recursively enumerable if and only if some deterministic Turing machine accepts it.

(b) A language is recursively enumerable if and only if some non-deterministic Turing machine accepts it.

Which of the following is true?

A: Only (a) is true.  
B: Both (a) and (b) are true.  
C: Neither (a) nor (b) is true.  
D: Only (b) is true.

**Question 2** (3pts, -1pt) Suppose a multi-tape deterministic Turing machine decides a problem $P$ in $O(n^3)$ time. Which of the following statements are true?

(a) There exists a single-tape deterministic Turing machine which decides $P$ in $O(n^3)$ time.

(b) There exists a single-tape deterministic Turing machine which decides $P$ in $O(n^8)$ time.

(c) There exists a single-tape deterministic Turing machine which decides $P$ in $O(n^9)$ time.

A: Only (b) is true.  
B: Only (a) is true.  
C: Only (c) is true.  
D: Only (b) and (c) are true.  
E: None of (a), (b) and (c) are true.  
F: All of (a), (b) and (c) are true.  
G: Only (a) and (c) are true.  
H: Only (a) and (b) are true.

**Question 3** (2pts, -1pt) Which of the following is the definition of the worst-case time complexity of a problem?

A: none of the above  
B: the worst-case time complexity of the best algorithm for solving that problem  
C: the worst-case time complexity of the worst known algorithm for solving that problem  
D: the worst-case time complexity of the best known algorithm for solving that problem  
E: the worst-case time complexity of the worst algorithm for solving that problem
Question 4  (3pts, -1pt) Consider the following statements:

(a) The class NP is closed under intersections of languages.

(b) The class NP is closed under concatenation of languages.

Which of the following is true?

A Only (b) is true.  C Only (a) is true.
B Both (a) and (b) are true.  D Neither (a) nor (b) is true.

Question 5  (3pts, -1pt) Let $L$ be a language such that a reduction exists from the set

$$\{M\#x \mid M \text{ loops on } x\}$$

to $L$. Consider the following statements:

(a) $L$ is not recursive.

(b) $L$ is recursively enumerable.

Which of the following are true?

A Both (a) and (b) are true.  C Neither (a) nor (b) is true.
B Only (b) is true.  D Only (a) is true.

Question 6  (2pts, -1pt) Suppose we have a $O(2^{\log n})$ algorithm for HAM-PATH (the Hamiltonian path problem). Does this mean that SAT (the boolean satisfiability problem) can be solved in time $O(2^{\log n})$?

A Yes  B No

Question 7  (3pts, -1pt) Let $A \subseteq \Sigma^*$ and $B \subseteq \Delta^*$ be two languages and let $f : \Sigma^* \rightarrow \Delta^*$ be a reduction from $A$ to $B$. Consider the following statements:

(a) If $B$ is not recursively enumerable then $A$ is not recursively enumerable.

(b) If $A$ is not recursively enumerable then $B$ is not recursively enumerable.

(c) $A$ is recursive if and only if $B$ is recursive.

Which of the following is true?

A Only (b) and (c) are true.  F All of (a), (b) and (c) are true.
B Only (a) and (b) are true.  G Only (a) is true.
C Only (a) and (c) are true.  H Only (c) is true.
D None of (a), (b) and (c) are true.
Question 8  (4pts, -1pt) Which of the following define a reduction from the Halting set to the set $E = \{M \mid \epsilon \in L(M)\}$?

(a) The reduction maps $M\#x$ to a Turing machine $M'$ which, on input $x'$: simulates $M$ on $x'$; if $M$ either accepts or rejects $x$, then $M'$ accepts $x'$.

(b) The reduction maps $M\#x$ to a Turing machine $M'$ which, on input $x'$: ignores $x'$ and simulates $M$ on $x$; $M'$ accepts $x'$ whenever $M$ accepts $x$.

(c) The reduction maps $M\#x$ to a Turing machine $M'$ which, on input $x'$: ignores $x'$ and simulates $M$ on $x$; if $M$ either accepts or rejects $x$, then $M'$ accepts $x'$.

A  Only (a) is a reduction.
B  Only (c) is a reduction.
C  None of the other options.
D  None of (a), (b) and (c) are reductions.
E  Only (b) is a reduction.

Question 9  (2pts, -1pt) Let CONNECTED be the problem of deciding whether a given directed graph is connected (that is, any node can be reached from any other node by travelling along the edges of the graph). Is CONNECTED in P?

A  Yes
B  Not known
C  No

Question 10  (3pts, -1pt) Let PATH be the problem of deciding whether a given graph contains a path between two given nodes. Is the following statement true or false? If PATH is NP-hard, then $P = NP$.

A  False
B  True

Question 11  (2pts, -1pt) Is the following true or false? If $f(n)$ is $\Theta(g(n))$, then $g(n)$ is $O(f(n))$.

A  True
B  False

Question 12  (3pts, -1pt) Consider the following statements:

(a) A language is recursive if and only if some non-deterministic Turing machine rejects it.

(b) A language is recursive if and only if some non-deterministic Turing machine accepts it.

(c) A language is recursive if and only if some non-deterministic decider accepts it.

Which of the following is true?

A  Only (b) and (c) are true.
B  Only (a) and (c) are true.
C  Only (a) is true.
D  All of (a), (b) and (c) are true.
E  Only (a) and (b) are true.
F  Only (c) is true.
G  Only (b) is true.
H  None of (a), (b) and (c) are true.
Question 13  (3pts, -1pt) Let \( A \subseteq \{0,1\}^* \) be a set. Which of the following statements is true:

A. The set \( \{ M \mid M \text{ is a Turing machine with input alphabet } \{0,1\} \text{ and } L(M) = A \} \) is always decidable.
B. The set \( \{ M \mid M \text{ is a Turing machine with input alphabet } \{0,1\} \text{ and } L(M) = A \} \) is always undecidable.
C. The set \( \{ M \mid M \text{ is a Turing machine with input alphabet } \{0,1\} \text{ and } L(M) = A \} \) is decidable for some choices of \( A \) and undecidable for others.

Question 14  (2pts, -1pt) Is the following true or false? \( 3^n = 2^{O(n)} \).
A. True  B. False

Question 15  (2pts, -1pt) Is the following true or false? \( n^2 = O(n \log^2 n) \).
A. True  B. False

Question 16  (3pts, -1pt) A path in a directed graph is called simple if it does not repeat any nodes. Let \( \text{LPATH} \) be the problem of deciding whether a given graph contains a simple path of length at least \( k \) between two given nodes. Is the following statement true or false? \( \text{LPATH} \) is NP-hard.
A. False  B. True

Question 17  (3pts, -1pt) Let \( L \) be a language such that a reduction exists from the Halting set to \( L \). Consider the following statements:

(a) \( L \) is recursive.
(b) \( L \) is recursively enumerable.

Which of the following are true?
A. Both (a) and (b) are true.
B. Neither (a) nor (b) is true.
C. Only (b) is true.
D. Only (a) is true.
Fill your registration id on the left circles and write your first name and last name below.

Firstname and lastname: ....................................................

Answers must be given exclusively on this sheet: answers given on the other sheets will be ignored. Please fill completely with your pen the box of the answer you want to select.

Question 1:  A  ■  C  D  
Question 2:  A  B  C  ■  E  F  G  H  
Question 3:  A  ■  C  D  E  
Question 4:  A  ■  C  D  
Question 5:  A  B  C  ■  
Question 6:  A  ■  
Question 7:  A  B  C  D  ■  E  F  G  H  
Question 8:  A  ■  C  D  E  
Question 9:  ■  B  C  
Question 10:  A  ■  
Question 11:  ■  B  
Question 12:  A  B  C  D  E  ■  G  H  
Question 13:  A  ■  C  
Question 14:  ■  B  
Question 15:  A  ■  
Question 16:  A  ■  
Question 17:  A  ■  C  D