1. Use diagonalisation to show that the membership problem is undecidable. That is, show that the following language is not recursive:

\[ MP \equiv \{ M \# x \mid x \in L(M) \}. \]

2. Prove that \( LP \equiv \{ M \# x \mid M \text{ loops on } x \} \) is not recursively enumerable.

3. Construct a reduction from \( LP \equiv \{ M \# x \mid M \text{ loops on } x \} \) to \( \{ M \mid L(M) = \emptyset \} \) to prove that the latter is not recursively enumerable.

4. Show that the language \( A \equiv \{ M \mid L(M) = L(a^*) \} \) is not recursive by constructing a reduction from HP.

5. Show that the language \( \{ M \mid M \text{ loops on some input} \} \) is not r.e.

6. Show that the following problems are undecidable:
   
   (i) given TMs \( M \) and \( N \), does \( L(M) = L(N) \)?  Hint: Use Question 4.
   
   (ii) given TMs \( M \) and \( N \), does \( L(M) \subseteq L(N) \)?  Hint: Use Question 3.
   
   (iii) given TMs \( M \) and \( N \), is \( L(M) \cap L(N) = \emptyset \)?  Hint: Use Question 3.