1. For each of the following questions, assume that $A$ is an NP problem. In each case, the question is "What would you tell the world?". In other words, what is the strongest or most interesting conclusion you can draw from this information?

(a) Suppose you prove a theorem showing that a lower bound for the running time of any algorithm to solve $A$ is $\Theta(2^n)$.

(b) Suppose you find a deterministic algorithm that solves $A$ in polynomial time.

(c) Suppose $A$ is NP-complete and you find a deterministic algorithm that solves $A$ in polynomial time.

(d) Suppose you prove that the satisfiability problem is reducible to $A$ in polynomial time.

2. Here, suppose that $A$ and $B$ are NP problems, and that there is a polynomial time reduction from $A$ to $B$. Again, what would you tell the world?

(a) Suppose you find a deterministic algorithm that solves $B$ in time $O(n\log n)$, and you know that $A$ is NP-complete.

(b) Suppose that the complexity of $A$ is $\Theta(n!)$.

3. Let DOUBLE\_SAT be the problem of deciding whether a boolean expression has at least two satisfying assignments. Show that DOUBLE\_SAT is NP-complete.

4. Show that if $P = NP$, then every non-trivial decision problem (that is, it has both yes instances and no instances) in $P$ is NP-complete.