COMP2210 CLASS TEST 2

Answers must be copied to the answer sheet or they will be ignored!
The test has 29 questions to complete in 45 minutes.
No documents allowed. The use of electronic calculators is forbidden.

We denote with \( \mathbb{N} \) the set of natural numbers including 0.

**Question 1**
Is \( \{a^n b^m \mid n, m \in \mathbb{N} \land n > m\} \cup \{a^n b^m \mid n, m \in \mathbb{N} \land n < m\} \) regular? (2pt)

A NO  
B YES

**Question 2**
Is \( \{a^n b^m \mid n, m \in \mathbb{N} \land n > m\} \cup \{a^n b^m \mid n, m \in \mathbb{N} \land n \leq m\} \) regular? (2pt)

A NO  
B YES

**Question 3**
Is \( \{a, b\}^* \setminus \{a^n b^n \mid n \in \mathbb{N} \land n > 0\} \) regular? (2pt)

A NO  
B YES

**Question 4**
Is there a regular language \( L \) such that \( L = L_1 \cup L_2 \) where both \( L_1 \) and \( L_2 \) are not regular languages? (2pt)

A YES  
B NO

**Question 5**
Is \( \{a^n b^n \mid n \in \mathbb{N} \land n > 0\} \cup L(a^* b^*) \) regular? (2pt)

A YES  
B NO

**Question 6**
Is \( \{a^n b^{2m} \mid n, m \in \mathbb{N} \land n, m > 0\} \) regular? (2pt)

A NO  
B YES

A string of parentheses is *balanced* when each left parenthesis has a corresponding right parenthesis, and the pairs are well-nested. For example, \((()())\) and \((())()\) are balanced while \((()\) and \()(()\) are not. Consider the context-free grammar \( G = (\{S\}, \{(),\}, P, S) \) where \( P \) is defined as follows:

\[
S \rightarrow (S) \mid SSSS \mid \epsilon
\]

Answer the following 3 questions:

**Question 7**
Is \( L(G) \) the set of all balanced parentheses? (2pt)

A NO  
B YES

**Question 8**
Is \( ((())()) \) a sentence of \( G \)? (2pt)

A NO  
B YES
Question 9  Is the following a derivation of $G$? (2pt)

\[ S \Rightarrow (S) \Rightarrow (SSSS) \Rightarrow ((S)S) \Rightarrow ((S)(S)) \Rightarrow (((S)(S))(S)) \Rightarrow (((S)(S))(S)) \Rightarrow (((S)(S))(S)) \]

A YES  B NO

End of this group.

Let $M$ be the pushdown automaton defined below.

<table>
<thead>
<tr>
<th>State</th>
<th>Input</th>
<th>Transition</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>a</td>
<td>a \cdot \bot</td>
</tr>
<tr>
<td>1</td>
<td>b</td>
<td>b \cdot \bot</td>
</tr>
<tr>
<td></td>
<td>c</td>
<td>\epsilon \cdot \bot</td>
</tr>
</tbody>
</table>

Answer the following 4 questions:

Question 10  Is $\{ a^n b^m \mid m, n \in \mathbb{N} \text{ and } m > n \}$ the language of all strings accepted by $M$ by final state? (2pt)

A YES  B NO

Question 11  Is $\{ a^n b^m \mid m, n \in \mathbb{N} \text{ and } m \geq n \}$ the language of all strings accepted by $M$ by empty stack? (2pt)

A YES  B NO

Question 12  Is the following an accepting computation of $M$ by final state? (1pt)

\[(0, abb, \bot) \rightarrow (0, bb, a \cdot \bot) \rightarrow (1, b, \bot) \rightarrow (1, \epsilon, \bot) \rightarrow (2, \epsilon, \epsilon)\]

A NO  B YES

Question 13  Is the following an accepting computation of $M$ by empty stack? (1pt)

\[(0, abb, \bot) \rightarrow (0, bb, a \cdot \bot) \rightarrow (1, b, \bot) \rightarrow (1, \epsilon, \bot)\]

A NO  B YES

End of this group.

Let $G = \{ N, \{ a, b, c \}, P, S \}$ be the context-free grammar where $P$ is defined as follows:

\[ S \rightarrow aSc \mid aA \mid B \]
\[ A \rightarrow aAc \mid aA \mid \epsilon \mid C \]
\[ B \rightarrow bBc \mid bC \]
\[ C \rightarrow bCc \mid bC \mid \epsilon \]

Answer the following 2 questions:
Question 14  Is $L(G) = \{wc^n | n \in \mathbb{N} \text{ and } w \in L(a^*b^*) \text{ and } |w| > n\}$? (2pt)

A YES  B NO

Question 15  Is $L(G) \subseteq \{a^nb^mc^k | m, n, k \in \mathbb{N} \text{ and } k \neq n+m\}$? (2pt)

A YES  B NO

End of this group.

Question 16  Is $\{a^n b^m | n, m \in \mathbb{N} \text{ and } n \geq m\} \cup \{a^n b^n | n, m \in \mathbb{N} \text{ and } n \leq m\}$ regular? (2pt)

A NO  B YES

Let $G = (N, \{a, b\}, P, S)$ be a context-free grammar with

$$P = \{(S, ABC), (S, aBb), (A, B), (B, C), (C, \epsilon), (C, A)\}$$

Answer the following 5 questions:

Question 17  Is $\{(S, ABC), (A, B), (B, C), (C, A)\}$ the set of all unit-production of $G$? (1pt)

A NO  B YES

Question 18  Is $G$ in Greibach normal form? (1pt)

A NO  B YES

Question 19  Is $\epsilon$ derivable from $S$ in $G$? (1pt)

A NO  B YES

Question 20  Is $B$ derivable from $S$? (1pt)

A NO  B YES

Question 21  Is $\{a^n b^n | n \in \mathbb{N}\} \subseteq L(G)$? (2pt)

A YES  B NO

End of this group.

Let $L = \{a^{2n}b^{2n} | n \in \mathbb{N}\}$. Consider the demon game. Suppose that for any $k > 0$ picked by the demon, we pick $x = a^k, y = a^kb^k$, and $z = b^k$. Answer the following 3 questions.

For all $u, v, w$ such that $y = uvw$ and $v \neq \epsilon$,

Question 22  is $xuv^0wz \notin L$? (2pt)

A NO  B YES
Question 23  is $xw^2wz \notin L$? (2pt)

A NO  B YES

Question 24  is $xw^3wz \notin L$? (2pt)

A YES  B NO

End of this group.

Question 25  Let $G$ be any context-free grammar in Chomsky normal form and in Greibach normal form. Is $L(G)$ regular? (2pt)

A NO  B YES

Question 26  Is $\{a^n b^m c^k \mid m,n,k \in \mathbb{N} \& m,n,k > 0 \& m = n\} \cap \{a^n b^m c^k \mid m,n,k \in \mathbb{N} \& m,n,k > 0 \& n = k\}$ regular? (2pt)

A YES  B NO

Let $G = \{N, \{a, b, c\}, P, S\}$ be the context-free grammar with

$$P = \{(S, aSc), (S, S'), (S', bS'c), (S', S), (S', \epsilon)\}$$

Answer the following 3 questions:

Question 27  Is $L(G) = \{we^n \mid n \in \mathbb{N} \& w \in L(a+b)^* \& |w| = n\}$? (2pt)

A YES  B NO

Question 28  Is $\{a^n b^m c^k \mid m,n,k \in \mathbb{N} \& n + m = k\} \subseteq L(G)$? (2pt)

A YES  B NO

Question 29  Is $L(G) \subseteq \{a^n b^m c^k \mid m,n,k \in \mathbb{N} \& n + m = k\}$? (2pt)

A YES  B NO

End of this group.