Learning Outcomes

By the end of this session you should be able to

... prove that CFL are closed under union, concatenation, Kleen star, and under intersection with regular languages
Closure properties - Union

• CFLs are closed under union.
  
  o If $G_1 = (N_1, \Sigma, P_1, S_1)$ and $G_2 = (N_2, \Sigma, P_2, S_2)$ are CFGs, with $N_1 \cap N_2 = \emptyset$, then let $G_3$ consist of:
    
    * $N_3 \overset{\text{def}}{=} N_1 \cup N_2 \cup \{S_3\}$, where $S_3 \notin N_1 \cup N_2$;
    
    * $P_3 \overset{\text{def}}{=} P_1 \cup P_2 \cup \{S_3 \rightarrow S_1, S_3 \rightarrow S_2\}$;
    
    * start nonterminal: $S_3$.
  
  o $L(G_3) = L(G_1) \cup L(G_2)$
Concatenation

- CFLs are closed under concatenation.
  - If $G_1 = (N_1, \Sigma, P_1, S_1)$ and $G_2 = (N_2, \Sigma, P_2, S_2)$ are CFGs, with $N_1 \cap N_2 = \emptyset$, then let $G_3$ consist of:
    - $N_3 \overset{\text{def}}{=} N_1 \cup N_2 \cup \{S_3\}$, where $S_3 \notin N_1 \cup N_2$;
    - $P_3 \overset{\text{def}}{=} P_1 \cup P_2 \cup \{S_3 \rightarrow S_1 S_2\}$;
    - start nonterminal: $S_3$.
  - $L(G_3) = L(G_1) L(G_2)$
• CFLs are closed under Kleene star.
  
  ° If $G_1 = (N_1, \Sigma, P_1, S_1)$ then let $G_2$ consist of:
    
    $\star N_2 \overset{\text{def}}{=} N_1 \cup \{S_2\}$, where $S_3 \notin N_1$;
    
    $\star P_2 \overset{\text{def}}{=} P_1 \cup \{S_2 \rightarrow S_1S_2, S_2 \rightarrow \epsilon\}$;
    
    $\star$ start nonterminal: $S_2$.
  
  ° $L(G_2) = L(G_1)^*$
Intersection with regular lang

- CFLs are closed under intersection with regular languages;
- Idea: Product construction with PDA and a DFA.
Learning Outcomes

You should be able to . . .

• . . . *prove* that CFL are closed under union, concatenation, Kleen star, and under intersection with regular languages