1. Convert the following NFA into a DFA using the subset construction. Show clearly which subset of the states of the NFA corresponds to each state of the DFA that you have constructed. Do not include states that are not reachable.

![NFA Diagram]

2. Convert the following NFAs into DFAs using the subset construction.

![NFA Diagrams]

3. Use the procedure from Lecture 3 in order to come up with a regular expression for the following NFA. You don’t need to follow the recursion to the end. Try to simplify at each step.

![NFA Diagram]

4. Give regular expressions for the following sets of strings over \{a, b\}. Aim for simplicity.

   a) strings with an even number of a’s;
   b) strings with an odd number of b’s;
   c) strings with an even number of a’s or an odd number of b’s.
   d) strings with an even number of a’s and an odd number of b’s;

5. Given an NFA \( M = (Q, \Sigma, \Delta, s, F) \), define a function \( \widehat{\Delta} : \Sigma^* \to 2^Q \) recursively as follows:

\[
\widehat{\Delta}(\epsilon) = \{s\} \\
\widehat{\Delta}(x\sigma) = \bigcup_{q \in \widehat{\Delta}(x)} \Delta(q, \sigma).
\]

Prove that \( x \in L(M) \) if and only if \( \exists f \in F. f \in \widehat{\Delta}(x) \).
6. Prove that the subset construction preserves the language accepted. That is, starting with any NFA $M$ and constructing the corresponding DFA $M'$ using the subset construction, show that $L(M) = L(M')$.

7. Prove that the construction in Lecture 2 taking an εNFA $M$ to an ordinary NFA $M'$ preserves the language accepted.