1. Use the pumping lemma to show that the language of all words over the alphabet \{0, 1\} containing the same number of 0’s and 1’s is not regular.

2. Given a language $L$ over an alphabet $A$, the complement of $L$ w.r.t. $A$ is defined to be all words over $A$ that are not members of $L$. The reverse of a language is the set of words which, if their letters were reversed, would give words in $L$.
   (a) Prove that if $L$ is a regular language then the complement of $L$ is also regular.
   (b) Prove that if $L$ is a regular language then so is the reverse of $L$.

3. Use the pumping lemma to show that any finite language is regular.

4. Construct a pushdown automaton that accepts (by final state) the set of strings in \{a, b\}∗ that have an equal number of a’s and b’s ($\{x \mid \#a(x) = \#b(x)\}$). Specify all the transitions. Simulate your automaton on $\epsilon$, $aaba$ and $abba$.

5. Given two PDAs, say $P_1$ and $P_2$, construct a PDA $P$ that accepts $L(P_1) \cup L(P_2)$.

6. Given a PDA $A$ (that accepts by final states), construct a PDA that accepts $L(A)^*$.

7. Consider the following context-free grammar $G$:
   
   \[
   \begin{align*}
   S & \rightarrow ABS \mid AB \\
   A & \rightarrow aA \mid a \\
   B & \rightarrow bA
   \end{align*}
   \]

   Which of the followings strings are in $L(G)$ and which are not? Provide derivations for those that are in $L(G)$. Explain the reasons for the strings that are not in $L(G)$.
   
   a) $aabaab$
   b) $aaaaba$
   c) $aabbba$
   d) $abaaba$

8. Define a CFG $G$ such that $L(G) = L(M)$ for any given DFA $M$.

9. Give a grammar with no $\epsilon$- or unit productions that generates the set $L(G) - \{\epsilon\}$ where $G$ is the grammar:
   
   \[
   \begin{align*}
   S & \rightarrow aSbb \mid T \\
   T & \rightarrow bTaa \mid S \mid \epsilon
   \end{align*}
   \]