1. Consider the labelled transition system below

\[
x_0 \xrightarrow{c} x_3 \\
\downarrow \quad \downarrow \\
\quad a \quad a \\
\quad x_1 \xrightarrow{c} x_2
\]

Give the set of traces from \( x_0, x_1, x_2, x_3 \).

2. Consider the labelled transition system below.

\[
y_0 \xrightarrow{a} y_1 \xrightarrow{a} y_2 \\
\xrightarrow{c} \quad \xrightarrow{c}
\]

(a) Are \( x_0 \) (from the previous question) and \( y_0 \) trace equivalent?

(b) Show that \( y_0 \) simulates \( x_0 \).

(c) Play the simulation game to show that \( x_0 \) does not simulate \( y_0 \).

3. Show that \( x_0 \) and \( y_0 \) in the labelled transition system below are bisimilar.

\[
x_0 \quad y_0 \\
\downarrow \quad \xrightarrow{a} \\
x_1 \quad x_1 \xrightarrow{a} \quad x_1
\]
4. Consider the two labelled transition system below.

(a) Show that \{(x_0, y_0), (x_1, y_1), (x_1, y_3), (x_3, y_3), (x_2, y_2), (x_2, y_4)\} is a simulation.

(b) Play the bisimulation game to show that \(x_0\) and \(y_0\) are not bisimilar.

5. Prove that a union of two simulations is a simulation.

6. Prove that simulation equivalence implies trace equivalences. That is, if \(x\) and \(y\) are simulation equivalent then they are capable of executing the same set of traces.