Response to nonperiodic excitation for single-degree-of-freedom systems.

- Previously:
  - Free-vibration: decaying sinusoid
  - Steady-state forced harmonic vibration

**Transient Response Due to Specified Forcing History:**

![Graph showing force F(t) and time t]

- Take a strip of width \( \Delta t \). Work out response due to this strip.
- "Sum" the response due to these strips.

Response due to an arbitrary excitation.

The magnitude of impulse (i.e. \( \int F(t) \, dt \)) due to a rectangular pulse of width \( \Delta t \) at time \( t = \tau \):

\[
F(\tau) \, \Delta t.
\]

since a force at time \( t = 0 \) with impulse \( \hat{F} \) is represented by

\[
\hat{F} \delta(t).
\]

The force corresponding to this rectangular pulse can approximately be written as

\[
\frac{F(\tau) \, \Delta t \cdot \delta(t-\tau)}{\text{impulse delta function}}.
\]

\[\text{The response due to this pulse for } t > \tau\]

\[
F(\tau) \, \Delta t \cdot h(t-\tau).
\]

The total response due to all the strips of width \( \Delta t \):

\[
x(t) \approx \sum F(t) \, \Delta t \cdot h(t-\tau).
\]

As \( \Delta t \to 0 \),

\[
x(t) = \int_{-\infty}^{t} F(\tau) h(t-\tau) \, d\tau
\]
For an oscillator at rest at $t=0$ and at $x=0$, the limit of integration can be brought forward to $t=0$.

$$x(t) = \int_{\tau=0}^{t} F(\tau) h(t-\tau) d\tau$$

Known as the Duhamel integral. "Dummy time variable"

Also known as the convolution of $F$ and $h$,

i.e.

$$x = F * h$$ for short.

We assumed:

- Linearity
- Causality.

$* = \text{convolution operator}$
Example: Pulse of finite duration $T$.

For the undamped case, the impulse response function $h(t)$ is given by

$$h(t) = \frac{1}{m\omega_n} \sin \omega_n t.$$
Note: The contribution due to the second term applies only for \( t \geq T \). And that due to the first term applies only for \( t > 0 \).

Hence:

\[
x(t) = 0, \quad t < 0
\]

\[
= \frac{F_0}{m \omega_n^2} (1 - \cos \omega_n t), \quad 0 \leq t \leq T
\]

\[
= \frac{F_0}{m \omega_n^2} \left[ (1 - \cos \omega_n t) - (1 - \cos \omega_n (t - T)) \right]
\]

\[
t > T.
\]