1. Equation of motion: \[ m \ddot{x} + c \dot{x} + kx = kx + cx \]

Insert: \[ \begin{align*}
  x &= xe^{\omega t} \\
  z &= ze^{\omega t}
\end{align*} \]

To get: \[ \frac{Z}{X} = \frac{\frac{1}{k} - i\omega c}{\omega^2 m + i\omega c} = H(\omega) \]

2. When fully loaded: \[ k = 350 \text{ N/m} \]

\[ m = 1000 \text{ kg} \]

\[ \delta = 0.5 \]

When empty \[ m = 250 \text{ kg} \]

We know \[ H(\omega) = \frac{Z}{X} = \frac{\frac{1}{k} - i\omega c}{(\omega^2 m + i\omega c)} \]

\[ \begin{align*}
  \left| \frac{Z}{X} \right| &= \frac{\sqrt{\frac{1}{k^2} + \omega^2 c^2}}{\sqrt{\omega^2 m^2 + \omega^2 c^2}} \\
  &= \frac{\sqrt{1 + \left(\frac{\delta}{\omega} \right)^2}}{\sqrt{\left(1 - \frac{\omega}{\omega_n}\right)^2 + \left(\frac{\delta}{\omega_n}\right)^2}} \\
\end{align*} \]

Excitation frequency: \[ f = \frac{v}{\lambda} \Rightarrow \text{any velocity} \ w = \frac{2\pi v}{\lambda} \]

\[ \begin{align*}
  f &= \frac{2\pi \times 100,000}{5 \times 3600} = 34.9 \text{ rad/s} \\
  \omega_n &= \sqrt{\frac{k}{m}} = \sqrt{\frac{350,000}{1000}} = 18.7 \text{ rad/s} \\
\end{align*} \]

\[ \frac{\omega}{\omega_n} = \frac{34.9}{18.7} = 1.87 \]

\[ \delta = 0.5 \Rightarrow \left| \frac{Z}{X} \right| = \frac{\sqrt{1 + (1.87)^2}}{\sqrt{1 - (1.87)^2 + (1.87)^2}} = 0.68 \]
Empty

$$w_n = \sqrt{\frac{m}{c}} = \sqrt{\frac{350,000}{250}} = 37.4 \text{ rad/s}$$

Frequency ratio $$\frac{\omega}{w_n} = \frac{34.9}{37.4} = 0.93$$

Damping ratio $$\zeta = \frac{c}{2\sqrt{m c}} = \frac{c}{2\sqrt{m c}}$$

but we don't know c!

However, $$\zeta$$ was 0.5 when fully loaded, when mass was 1800 kg, now it is 0.4 so $$\zeta$$ will be clamped

$$\Rightarrow \zeta = 0.4$$

Then $$\frac{\omega}{w_n} = \sqrt{\frac{1 + (1.87)^2}{(1 - (0.93)^2)^2 + (1.87)^2}} = 1.13$$

3. Expand the disturbing force $$F(t)$$ shown into its Fourier series. From data book

$$F(t) = \frac{2F_0}{\pi} \left( \sin \omega_0 t - \frac{1}{3} \sin 3\omega_0 t + \frac{1}{5} \sin 5\omega_0 t - \cdots \right)$$

4. $$\frac{\omega}{w_n} = 0.8, \quad \zeta = 0.2$$

For $$F(t)$$ applied directly to the mass, the magnification factor

$$\left| \frac{X}{X_0} \right| = \frac{1}{\sqrt{\left(1 - \left(\frac{\omega}{w_n}\right)^2\right)^2 + \left(2\zeta \frac{\omega}{w_n}\right)^2}}$$

where $$X_0 = \frac{F_0}{K} = \text{static displacement}$$

First term in series $$\left| \frac{X}{X_0} \right| = \frac{1}{\sqrt{(1 - 0.8)^2 + (2 \times 0.2 \times 0.8)^2}} = 2.076$$
Second term: 
\[ \left| \frac{X}{X_0} \right| = \frac{1}{\sqrt{1 - (0.5)^2}} = 0.707 \]

Third term: 
\[ \left| \frac{X}{X_0} \right| = \frac{1}{\sqrt{1 - (0.25)^2}} = 0.632 \]

Divide through by largest to get ratio:

\[ \frac{0.707}{0.632} : (-\frac{1}{2})(0.707) : (\frac{1}{3})(0.632) \]

\[ = 1 : -0.143 : 0.03 \]

5a) Supporting springs have stiffness \( k = \frac{mg}{L} \)

\[ = \frac{1000 \times 9.81}{0.02} = 490500 \text{ N/m} \]

Natural frequency of system \( \nu_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{490500}{1000}} = 22.15 \text{ rad/s} \)

Modal operates at \( 1200 \nu_n = 400 \pi \text{ rad/s} \)

Frequency ratio \( \frac{\nu}{\nu_n} = \frac{400\pi}{22.15} = 5.7 \)

Out of balance force \( F(t) = mr^2 \sin \omega t = 1600\pi^2 \text{ Nm} \)

For \( c = 0, s = 0 \), from lectures:

\[ \left| \frac{F_t}{F_o} \right| = \sqrt{\frac{1}{1 - (\frac{L}{\nu_n})^2}} = \frac{1}{1 - (\frac{L}{\nu_n})^2} = \frac{1}{5.7^2 - 1} = 0.032 \]

\( F_o = 1600\pi^2 \), therefore \( |F_t| = 501.5 \text{ N} \)

b) We know the force applied to the block has magnitude \( F_o = 1600\pi^2 \)

From lectures:

\[ \left| \frac{X}{X_0} \right| = \frac{X}{X_0} = \frac{1}{\sqrt{1 - (\frac{L}{\nu_n})^2}} = \frac{1}{|1 - (\frac{L}{\nu_n})^2|} = 0.032 \]
\[ x_0 = \frac{F_0}{k} = \frac{1600\pi^2}{440,500} = 0.032 \text{ m} \]

\[ x = 0.032 \times 0.032 = 1.04 \text{ mm} \]

3) Wall concrete block, \( m = 3500 \text{ kg} \), \( n = \frac{mg}{g} = \frac{3500 \times 9.81}{0.02} = 171675 \text{ N/m} \)

\[ \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{171675}{3500}} = 22.15 \text{ rad/s} \]

\( x \) unchanged, \( F \) unchanged

Magnification factor \( \left| \frac{x}{x_0} \right| \) unchanged, but

\[ x_0 = \frac{F_0}{k} = \frac{1600\pi^2}{1716750} = 0.009 \]

\[ |x| = 0.009 \times 0.032 = 0.3 \text{ mm} \]