1) Show that the response of a damped single degree of freedom system to a step of amplitude $F_0$ with the initial conditions $x(0) = \dot{x}(0) = 0$, is

$$x(t) = \frac{F_o}{k} \left[ 1 - \frac{e^{-\zeta \omega_n t}}{\sqrt{1 - \zeta^2}} \cos \left( \sqrt{1 - \zeta^2} \omega_n t - \phi \right) \right]$$

where

$$\tan \phi = \frac{\zeta}{\sqrt{1 - \zeta^2}}$$

2) Determine the response of an undamped single degree of freedom system to a unit ramp input.

3) Show that the time $t_p$ corresponding to the peak response for the impulsively excited spring-mass-damper system is given by the equation

$$\tan \left( \sqrt{1 - \zeta^2} \omega_n t_p \right) = \frac{1 - \zeta^2}{\zeta}$$

4) Determine the peak displacement for the impulsively excited spring-mass-damper system, and show that it can be expressed in the form

$$\frac{x_{peak} \sqrt{k m}}{F_0} = \exp \left( - \frac{\zeta}{\sqrt{1 - \zeta^2}} \tan^{-1} \left( \frac{\sqrt{1 - \zeta^2}}{\zeta} \right) \right)$$

5) Show that the time $t_p$ corresponding to the peak response of the damped spring-mass system excited by a step force $F_0$ is

$$\omega_n t_p = \frac{\pi}{\sqrt{1 - \zeta^2}}$$

6) For the system of Problem 5), show that the peak response is equal to

$$\left( \frac{x_{peak}}{F_0} \right)_{max} = 1 + \exp \left( - \frac{\zeta \pi}{\sqrt{1 - \zeta^2}} \right)$$