Answer THREE questions.

A formula sheet is provided

University approved calculators MAY be used.
Question 1  Consider the circuit depicted in Figure 1.

![Circuit Diagram]

**Figure 1: The circuit for Question 1**

(a) Can we apply Bashkow’s algorithm in order to write state equations for the circuit in Figure 1? Justify your answer.

(3 marks)

(b) How would you select a set of state variables for the circuit in Figure 1?

(3 marks)

(c) Write state equations for the circuit in Figure 1, showing at each step how you arrive at your conclusions.

(10 marks)

(d) Identify the state matrix and the input matrix in the state equations you have written.

(3 marks)

(e) Assume now that the values of the components are $R = 1 \, \Omega$, $C = 1 \, F$, $L = 5 \, H$. Find the transfer function of the circuit from the current source $I(\cdot)$ (input) to the voltage $v_C$ across the capacitor.

(8 marks)

(f) Assume that a constant current equal to $1 \, A$ is applied at the source, and that the initial current in the induction $i_L(0) = 0 \, A$, and that
the initial voltage across the capacitor $v_C(0) = 0 \, V$. Find the (approximate) value of the voltage across the capacitor when $t$ takes on very large values (i.e. $t \to \infty$).

(6 marks)
Question 2  Consider the circuit depicted in Figure 2.

(a) Can we apply Bashkow’s algorithm in order to write state equations for the circuit in Figure 2? Justify your answer.

(3 marks)

(b) How would you select a set of state variables for the circuit in Figure 2?

(3 marks)

(c) Write state equations for the circuit in Figure 2, showing at each step how you arrive at your conclusions.

(10 marks)

(d) Identify the state matrix and the input matrix in the state equations you have written.

(3 marks)

(e) Assume now that the values of the components are as follows: \( R_1 = 1 \ \Omega, \ R_2 = 1 \ \Omega, \ C = 1 \ F, \ L = 1 \ H. \) Find the transfer function of the circuit from the voltage source \( E(t) \) (input) to the current in the inductor.

(8 marks)
(f) Assume that a constant voltage equal to 1 V is applied at the source, and that the initial current in the induction $i_L(0) = 0 \, A$, and that the initial voltage across the capacitor $v_C(0) = 0 \, V$. Find the (approximate) value of the current in the inductor when $t$ takes on very large values (i.e. $t \to \infty$).

(6 marks)
Question 3  Consider the circuit depicted in Figure 3, where $R_1 =$ $\begin{array}{c}
E_2 \\
\quad R_2 \\
R_1 \\
I_1 \\
\quad R_3 \\
R_4 \\
R_5 \\
\end{array}$

Figure 3: The circuit for Questions 1.a-1.e

$R_1 = 5 \, \Omega$, $R_2 = 4 \, \Omega$, $R_3 = 3 \, \Omega$, $R_4 = 6 \, \Omega$, $R_5 = 2 \, \Omega$, $I_1 = 4 \, A$, $E_2 = 40 \, V$. Enumerate the branches as in Figure 4, and assume that the tree consisting of branches 1 and 2 (solid line) has been selected.

Figure 4: The tree for Questions 1.a-1.e

(a) What is the minimal number of independent currents necessary in order to write down equations for all the currents in the circuit? Please justify your answer.

(3 marks)

(b) Write down the loop (tie-set) matrix $L$ for the tree selected in Figure 4.

(4 marks)
(c) Write down the branch currents in terms of the loop currents; express \textit{in matrix form} how these equations can be written.

\textit{(4 marks)}

(d) Write down the loop impedance matrix $Z$ for this circuit.

\textit{(4 marks)}

(e) Find \textit{numerical values} for the loop currents, explaining clearly how you computed them.

\textit{(5 marks)}

(f) Now use the same circuit as in Figure 3, but:

1.f.a Assume that the voltage generator has been substituted with a short circuit as in Figure 5. Find \textit{numerical values} for the loop currents in this case, using the same tree considered in Figure 4.

**********Question 3 continues on the next page**********
1.f.b Assume that the current generator has been substituted with a open circuit as in Figure 6. Find *numerical values* for the loop currents in this case, using the same tree considered in Figure 4.

1.f.c Explain in detail how the currents found answering Questions 1.f.a and 1.f.b relate to the currents found answering Question 1.e. Can you formulate a general principle encompassing situations such as these?

(13 marks)
Question 4

(a) Derive the line transmission equations:

\[
\frac{\partial^2 V}{\partial x^2} = ZYV \\
\frac{\partial^2 I}{\partial x^2} = ZYI
\]

where \( Z, Y \) are the per-length impedance and per-length admittance to ground, \( V \) and \( I \) are the voltage and current phasors, and \( x \) is the distance along the line from the load. \( 11 \) marks

(b) Show that

\[
V(x) = V_1 \exp(\gamma x) + V_2 \exp(-\gamma x) \\
I(x) = I_1 \exp(\gamma x) - I_2 \exp(-\gamma x)
\]

are solutions of the line transmission equations for a suitable expression for \( \gamma \) which you should determine. \( 6 \) marks

(c) Define the characteristic impedance \( Z_0 \). Show

\[
Z_0 = \sqrt{\frac{Z}{Y}},
\]

and

\[
Z_0 = \frac{Z}{\gamma}.
\]

\( 6 \) marks

(d) Suppose the line is terminated at \( x = 0 \) by a load equal to the characteristic impedance. By considering the equation

\[
\frac{\partial V}{\partial x} = -ZI,
\]

show that that \( V_1 = 0 \). \( 10 \) marks
Question 5

(a) Explain how the two Wattmeter technique works. Your explanation should include reasoning as to why the sum of the two Wattmeter readings equates to total power and should include suitably labeled diagrams. (10 marks)

(b) The two Wattmeter approach is applied to a three phase three wire 120V ABC system. The Wattmeters are connected to lines A and C, and read $W_A = 920W$, $W_C = 460W$ respectively. Find the impedance of the balanced Δ-connected load. (20 marks)

(c) What would the impedance of a balanced star connected load have to be in order to obtain the same Wattmeter readings? (3 marks)