Recap

- We have introduced the error probability plane and bandwidth efficiency plane and their relationships.
- We have discussed various trade-offs in designing digital communications systems, which can be broadly classified as
  - bandwidth limited system
  - power limited system
- We next discuss design principles using examples and introduce a joint coding and modulation scheme - Trellis Coded Modulation.
Choose the modulation scheme that meets: $R = 9.6$ kbits/s and $P_b \leq 10^{-5}$ when $W = 4$ kHz, $E_b/N_0 = 13.2$ dB.

1. determine the use of MPSK since $W < R \Rightarrow$ bandwidth limited.
2. determine the smallest $M$ so that $W_{min} \leq W \Rightarrow M = 8$.
3. determine if $P_b \leq 10^{-5}$ when $M = 8$ is satisfied \(\Rightarrow \) yes.

(since the threshold $E_b/N_0$ for achieving $P_b \leq 10^{-5}$ is 13 dB)
Choose the modulation scheme that meets: \( R = 9.6 \text{ kbits/s} \) and \( P_b \leq 10^{-5} \) when \( W = 45 \text{ kHz}, \ E_b/N_0 = 8.2 \text{ dB} \).

1. determine the use of MFSK since a) \( W \gg R \Rightarrow \) not bandwidth limited b) \( E_b/N_0 = 8.2 \text{ dB} \) cannot support any MPSK scheme \( \Rightarrow \) power limited.

2. determine the largest \( M \) so that \( W_{\text{min}} \leq W \Rightarrow M = 16 \).

3. determine if \( P_b \leq 10^{-5} \) when \( M = 16 \) is satisfied \( \Rightarrow \) yes. (since the threshold \( E_b/N_0 \) for achieving \( P_b \leq 10^{-5} \) is 8.1 dB)
## Bandwidth and Power Limited System Design

Choose the modulation scheme that meets: $R = 9.6$ kbits/s and $P_b \leq 10^{-5}$ when $W = 4$ kHz, $E_b/N_0 = 11$ dB.

1. determine the use of MPSK since $W < R \Rightarrow$ bandwidth limited.
2. determine the smallest $M$ so that $W_{\text{min}} \leq W \Rightarrow M = 8$.
3. determine if $P_b \leq 10^{-5}$ when $M = 8$ is satisfied $\Rightarrow$ no $\Rightarrow$ power limited $\Rightarrow$ we need channel coding.

<table>
<thead>
<tr>
<th>$M$</th>
<th>$k$</th>
<th>$R$ (bits/s)</th>
<th>$R_s$ (syms/s)</th>
<th>MPSK $W_{\text{min}}$ (Hz)</th>
<th>MPSK $E_b/N_0$ (dB) $P_b = 10^{-5}$</th>
<th>MFSK $W_{\text{min}}$ (Hz)</th>
<th>MFSK $E_b/N_0$ (dB) $P_b = 10^{-5}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>9600</td>
<td>9600</td>
<td>9600</td>
<td>9.6</td>
<td>19,200</td>
<td>13.4</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>9600</td>
<td>4800</td>
<td>4800</td>
<td>9.6</td>
<td>19,200</td>
<td>10.6</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>9600</td>
<td>3200</td>
<td>3200</td>
<td>13.0</td>
<td>25,600</td>
<td>9.1</td>
</tr>
<tr>
<td>16</td>
<td>4</td>
<td>9600</td>
<td>2400</td>
<td>2400</td>
<td>17.5</td>
<td>38,400</td>
<td>8.1</td>
</tr>
<tr>
<td>32</td>
<td>5</td>
<td>9600</td>
<td>1920</td>
<td>1920</td>
<td>22.4</td>
<td>61,400</td>
<td>7.4</td>
</tr>
</tbody>
</table>
Continued

1. **coding rate**: determine the maximum allowable bandwidth expansion ⇒ 25% (when $M = 8$, $W_{\text{min}} = 3.2 \text{ kbits/s}$ and $W = 4 \text{ kHz}$) ⇒ the coding rate should be higher than $4/5$.

2. **coding gain**: determine the minimum required coding gain ⇒ $13 - 11 = 2 \text{ dB}$.

3. **complexity**: the simplest code satisfying the coding gain and coding rate ⇒ $(63,51)$ BCH.
Instead of sending a symbol formed by \( k \) bits (say QPSK), we introduce one additional parity bit using convolutional code and send the resultant symbol formed by \( k + 1 \) bits (say 8-PSK).

- Bit 2 is fed directly to the 8PSK signal mapper.
- Bit 1 is encoded by a two-stage four-state half-rate CC.
- The CC adds the parity bit, Bit 0, to the sequence.
Euclidean distance amongst constellation points is increased at every partitioning step. Note the unprotected Bit 2 is assigned with $d_2$. 

(Bit 2, Bit 1, Bit 0)

$\begin{align*}
\text{Bit 0 = 0} & : 01, 000, 110 \\
\text{Bit 0 = 1} & : 01, 000, 111 \\
\text{Bit 1 = 0} & : 11, 110, 111 \\
\text{Bit 1 = 1} & : 01, 000, 001 \\
\text{Bit 1 = 0} & : 01, 000, 111 \\
\text{Bit 1 = 1} & : 01, 000, 001 \\
\end{align*}$
Parallel trellis transitions are associated with phasors having the maximum possible distance ($d_2$) between them, which are phasor points in the subsets (0,4), (1,5), (2,6) and (3,7).

All four-state transitions originating from, or merging into, any one of the states are labelled with phasors having a distance of at least $d_1 = \sqrt{2}$ between them. These are the phasors belonging to subsets (0,2,4,6) or (1,3,5,7).
The free distance is the minimum of the Euclidean distances between the phasors labelling the parallel branches and the distances between trellis paths diverging and re-merging after a number of consecutive trellis transitions.

- The Euclidean distance between the phasors associated with the parallel branches is $d_2 = 2$. The distance between the diverging trellis paths of 0-0-0 and 2-1-2 is $d_1 - d_0 - d_1$.
- This diverging path has the shortest accumulated Free Euclidean Distance (FED) over all the remerging paths of the trellis. So, the free distance is given by $d_{\text{free}} = \min\{d_2; \sqrt{d_1^2 + d_0^2 + d_1^2}\} = 2$.
- By contrast, the free distance of the un-coded QPSK is $\sqrt{2}$. 

▶
Summary

- We have discussed three designs for achieving different system requirements: bandwidth limited, power limited and double limited.
- We have briefly introduced the TCM, its encoding, principle and capability. This is a classic example of coded modulation, which is widely used in telephone line.
- The decoding of TCM uses symbol-based MAP detection.