Recap

- We have introduced the basic idea of spread spectrum technique and in particular the DSSS system.
- Spread spectrum technique exhibits many benefits, such as the interference suppression.
- We now discuss the Frequency Hopping spread spectrum system.
- It was invented back in 1941 by Hedy Lamarr, an Austrian-born Hollywood actress.
In conventional MFSK modulator, fixed carrier frequency is used and each tone represents one of the M symbols.

In FH/MFSK system, the carrier frequency is shifted randomly over a hopping bandwidth $W_{ss}$, which is much wider than the required bandwidth $W$. Hence the **processing gain** is $W_{ss}/W$.

The hopping bandwidth $W_{ss}$ and the minimum frequency spacing $\Delta f$ determine the number of spreading chips required, where we have $n = \lceil \log_2 (W_{ss}/\Delta f) \rceil$.

A reverse operation is applied at the receiver, where the signal is firstly de-hopped and then demodulated.
Data rate $R = 150$ bits/s, using 8-FSK, sym rate $R_s = 50$ syms/s (sym duration $T_s = 20$ ms) and tone spacing $\Delta f = 50$ Hz. The frequency is hopped once per symbol, i.e. 50 hops/s. At each hopping transition, the center frequency in dashed line is hopped to a new place, while the solid line denotes the tone.
FHSS - Example

- The frequency is hopped four times per symbol, i.e. 200 hops/s.
- The 20 ms symbol duration is divided into four 5 ms chip duration.
- The duration of each FSK tone is 5 ms, then $\Delta f = 200$ Hz.
Fast FH means several hops per symbol - chip rate is hop rate. For example, 1 symbol duration = 1/30 s, 4 hops for each symbol, then chip duration is 1/120 s.

Slow FH means several symbols per hop - chip rate is symbol rate. For example, 1 symbol duration = 1/30 s, 3 symbols for each hop, then hop duration is 1/10 s and chip duration is 1/30 s.
Example

Considering a FHSS system using MFSK modulation. Assume 8-FSK is used and the data rate is $R = 1.2$ kbits/s. Assume also the hopping rate is $R_h = 2$ khops/s and the hopping bandwidth $W_{ss} = 120$ MHz.

- Is the system fast or slow hopping? What is the chip rate and what is the minimum frequency spacing?
- How many chips are required and what is the processing gain?

Data rate is $R = 1.2$ kbits/s, using 8-FSK, symbol rate is $R_s = 0.4$ ksym/s. Hopping rate is $R_h = 2$ khops/s, so this is fast hopping and chip rate equals to hopping rate $R_c = 2$ kchips/s. Hence, the minimum frequency spacing is $\Delta f = 2$ kHz.

The number of chips are $n = \lceil \log_2 (W_{ss}/\Delta f) \rceil = 16$ chips. The processing gain is $W_{ss}/R = 100,000$ or 50 dB.
Each user is assigned a user-specific spreading code $g_i(t)$, where multiple users can access the system at the same time and frequency.

- Multi-user interference is eliminated when orthogonal spreading code is employed where $\int_0^T g_i(t)g_j(t)dt = 0$ for $i \neq j$.
- When spreading code is not perfectly orthogonal, cross-correlation between $g_i(t)$ and $g_j(t)$ plays important role.
Walsh Hadamard code is a class of orthogonal codes. In general, for \( k \geq 2 \), it can be constructed by

\[
H(2^k) = \begin{bmatrix}
H(2^{k-1}) & H(2^{k-1}) \\
H(2^{k-1}) & -H(2^{k-1})
\end{bmatrix}; \quad H(2) = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}
\]  

(1)

When the spreading code length is \( N = 2^k = 4 \), a total of four users can access the system simultaneously using one of the codes:

\( g_1 = [1, 1, 1, 1], g_2 = [1, -1, 1, -1], g_3 = [1, 1, -1, -1], g_4 = [1, -1, -1, 1] \). Since \( g_i g_j^T = 0, i \neq j \), interference is eliminated.
The received signal may be expressed as

\[ r(t) = Ax(t)g(t) \cos \omega_0 t + \alpha Ax(t - \tau)g(t - \tau) \cos(\omega_0 t + \theta) \] (2)

Correlating the received signal with \( g(t) \), we get

\[ z(t = T) = \int_0^T [Ax(t)g^2(t) \cos \omega_0 t + \alpha Ax(t - \tau)g(t - \tau)g(t) \cos(\omega_0 t + \theta)] 2 \cos \omega_0 t dt \] (3)

Since \( g^2(t) = 1 \) and \( g(t - \tau)g(t) \approx 0 \) for \( \tau > T_c \), interference is eliminated and \( z(t = T) = \int_0^T 2Ax(t) \cos^2 \omega_0 t = Ax(T) \).
Summary

- FHSS is an important technique that transmits at a much wider bandwidth $W_{ss}$ than the required bandwidth $W$, where the associated processing gain is $W_{ss}/W$.

- Following concepts are especially important in charactering slow and fast frequency hopping:
  - hopping bandwidth and minimum frequency spacing.
  - hop duration, symbol duration, bit duration and chip duration.

- DS/CDMA is an important application of DSSS system, which allows multiple users to access the system with same time and frequency. It is also capable of combating multiuser interference and multipath interference.