Life-Time Testing

The topic of reliability is of major importance in electrical industries.
One method of measuring the reliability of a device is to test a batch of items over an extended period of time and to note the failure times (This process is called life testing).

It is desirable to carry out the tests under the same conditions as would be met in practical use.

⇒ **Problem**: require an unrealistic waiting time before useful results become available.

**Possible solution**: accelerating life test.
Two ways of accelerating life testing:
-- Compressed-time testing: used more intensively than usual but without changing the stress levels
-- Advanced-stress testing: subject the device to higher stress level than would normally apply.

**Caution:** Data obtained from accelerated life testing is much less reliable than that obtained under normal conditions and should always be treated with suspicion. Understanding the failure mechanisms and mathematical description are therefore important.
Failure Distributions

- Failure is a random event, i.e. statistical in nature
- A proper data analytical tool is required
- Various probability distributions:
  - Uniform distribution
    \[
    f(x) = \begin{cases} 
    \frac{1}{b-a}, & a < x < b \\ 
    0, & \text{otherwise}
    \end{cases}
    \]
  - Normal (Gaussian) distribution
    \[
    f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{x-\mu}{\sigma} \right)^2}, \quad -\infty < x < \infty
    \]
  - Weibull distribution
    \[
    f(x) = \frac{\alpha}{\beta} x^{\alpha-1} e^{-(x/\beta)\alpha}, \quad x \geq 0
    \]
Weibull Distribution

There is nothing god given about the Weibull distribution. There are other reliability distributions - all invented before Weibull. But it does fit, experimentally, a very wide variety of phenomena.

Weibull wrote a famous paper demonstrating it fits the size distribution of beans, the height distribution of the population on an island and so on, including failure of steels.
Weibull Analysis

This involves three steps

“Massaging” the data point.

Plotting the massaged data point in a “double ln” plot to get the Weibull parameters.

Learning on how to put confidence limits on the results.
Massaging the data point

The data are plotted in increasing sequence (ranking low to high)

The equivalent of our failure percentage becomes approximately

\[ \hat{F} = \frac{i - 0.3}{n + 0.4} \]

This will do for most engineering problems. One can do this better by looking up the F distribution.
Once you have done the ranking, it is just plotting:

\[
F(x) = 1 - e^{-\frac{(x/\lambda)^k}{\ln(1 - F(x))}} = \frac{(x/\lambda)^k}{\ln(1 - F(x))}
\]

\[
\ln\left(\frac{-\ln(1 - F(x))}{\ln x - \ln \lambda}\right) = 'y'
\]

And, from the double ln plot to extract the values for $K$ and $\lambda$

So here is the plot:
Weibull Distribution for Dielectric Failures

The Weibull distribution is applicable where the occurrence of an event in any part of an object means failure of the object as a whole.

It has been used successfully to describe among other things, fatigue failures, vacuum tube failures and dielectric breakdown and is the probably the most popular model for describing failure times.

The survival probability $P(t, E)$ is given by

$$P(t, E) = \exp[-\left(\frac{t}{t_0}\right)^a \left(\frac{E}{E_0}\right)^b]$$

where $a$, $b$, $t_0$ and $E_0$ are the Weibull parameters for a particular insulation (with particular dimensions).
For a fixed $E$, a plot of $\log[\log(P(t))]$ vs $\log(t)$ gives parameter $a$.

For a fixed $t$, a plot of $\log[\log(P(t))]$ vs $\log(E)$ gives parameter $b$.

For a constant probability of survival we have

$$tE^N = \text{const}.$$ 

known as the inverse power law or life function.

Relationship between $N$ and $a$ and $b$:

$$N = \frac{b}{a}.$$
How to apply the law?

This law is utilised by maintaining a constant stress on the testing object and measuring time to failure.

Life under service conditions is obtained by extrapolating the straight line resulting from the plot of Log(E) versus log(t).

This assumes that the same mechanism which has operated at high stresses operates at the service stress.
Health expert says that stress can kill people.

Insulation expert says that higher electric stress (field) can deteriorate insulation material and reduce its life span.

Accelerating ageing tests are necessary to establish N and the constant in the inverse power law.

Providing the same mechanisms involved in ageing processes, the inverse power law allow us to predict service life of XLPE cables.

Data collected from the literature for high voltage AC XLPE cables with different conductor sizes and insulation thicknesses.
The imperfections occur in a random manner. The more volume tested, the lower the survival probability. To extend the model tests to an operating system volume effect has to be considered

\[ P(t, E) = \exp\left[ -\left(\frac{t}{t_0}\right)^a \left(\frac{E}{E_0}\right)^b \frac{V}{V_0}\right] \]

In the case of cable

\[ P(t, E) = \exp\left[ -\left(\frac{t}{t_0}\right)^a \left(\frac{E}{E_0}\right)^b \frac{lr^2}{l_0 r_0^2}\right] \]

For the same survival probability

\[ \left(\frac{E}{E_0}\right) = \left[ \frac{l_0}{l} \left(\frac{r_0}{r}\right)^2 \right]^{1/b} \]
where

$E$ and $E_0$ are the operating cable and test model stresses

$l$ and $l_0$ are the operating cable and test model lengths

$r$ and $r_0$ are the operating cable and test model radii

Note: Once $a$ and $b$ for a particular cable are known and experiments revealing one set of values, $t_0$ and $E_0$, for a cable of known dimensions, it is possible to calculate the survival probability for any other cable of the same material of insulation.
Example: 100m samples of a power cable have been subjected to ageing tests under different electric stresses. Failure times at each stress have been analysed using Weibull distribution and characteristic times obtained as a function of the applied stress are as follows:

<table>
<thead>
<tr>
<th>E (kV/mm)</th>
<th>30</th>
<th>25</th>
<th>20</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>t (hours)</td>
<td>0.659</td>
<td>5.8</td>
<td>86</td>
<td>2700</td>
</tr>
</tbody>
</table>

A ramp test on the samples indicates that the value of ‘a’ the life distribution constant is equal to 0.5.

Determine the life function of the cable and the value of “b” the stress distribution constant.

Estimate the mean electric stress which may be applied to a 10 km length if it is to be used continuously under normal condition (survival probability of 90% for 40 years).

Calculate the survival time if the cable length is doubled.
Laboratory Test Procedures

The most frequently occurring overvoltages on electrical systems and apparatus originate in lightning and switching overvoltages.

The most laboratory tests are conducted under standard lightning impulse voltages and switching surge voltages.
Gaussian cumulative distribution function

\[ P(V) = \frac{1}{\sigma \sqrt{2\pi}} \int_{-\infty}^{V} e^{-\frac{(V-V_{50})^2}{2\sigma^2}} \, dx \]

\[ z = \frac{V - V_{50}}{\sigma} \]

\[ P(z) = \frac{1}{\sigma \sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{z^2}{2}} \, dz \]
Three general testing methods have been accepted:

- Multi-level method
- Up and down method
- Extended up and down method
1. Multi-level test method

In this method the procedure is:

- choose several test voltage levels,
- apply a pre-specified number of shots at each level (n),
- count the number ($x$) of breakdowns at each voltage level,
- plot $p(V) (x_j/n)$ against $V$ (kV),
- draw a line of best fit on a probability scale,
- from the line determine $V_{50}$ at $P(V) = 50$ per cent,
- and $\sigma = V_{50\%} - V_{16\%}$
• Probability of breakdown distribution

![Graph showing probability distribution]

• The advantage of this method:
  - it does not assume normality of distribution.

• The disadvantage:
  - it is time consuming, i.e. many shots are required.
2. *Up and down method*

- In this method a starting voltage ($V_j$) close to the anticipated flashover value is selected.
- Then equally spaced voltage levels ($\Delta V$) above and below the starting voltage are chosen.
- The first shot is applied at the voltage $V_j$.
- If breakdown occurs the next shot is applied at $V_j - \Delta V$.
- If the insulation withstands, the next voltage is applied at $V_j + \Delta V$.
- The step is repeated for many shots.
• The sequential procedure of testing:

![Diagram of sequential procedure]

- This method has the advantage that it requires relatively few shots.
- The disadvantage is that it assumes normality and is not very accurate in determining $\sigma$. 

$V_o = \text{lowest level at which a shot is applied}$
Determination of 50% and 10% flashover voltage

• After number of flashover tests, the 50% flashover voltage is calculated using the frequency of occurrence $n_i$ with which the corresponding voltage $V_i$ was tested.

$$V_{50} = \frac{1}{n} \sum_{i=1}^{n} n_i \times V_i$$

• The first valid voltage is the one when the breakdown behaviour changes for the first time.

• The 10% flashover:

$$V_{10} = V_{50} \times (1 - 1.3z) = V_{50} \times 0.96$$
3. The extended up and down method

- This method is also used in testing self-restoring insulation.
- It can be used to determine discharge voltages corresponding to any probability $p$.
- A number of impulses are applied at a certain voltage level.
- If none causes discharge, the voltage is increased by a step $\Delta V$ and the impulses are applied until at least one causes breakdown.
- Then the voltage is decreased.
The extended up and down method procedure
• The number \( n \) is determined such that a series of \( n \) shots would have 50 percent probability of giving at least one flashover. The 50 per cent probability of discharge is given by

\[
0.5 = 1 - (1 - p)^n
\]

or

\[
n = -\frac{0.693}{\ln(1 - p)}
\]

• The value \( n = 7 \) impulses per voltage level is often used as it allows the determination of 10 per cent discharge voltage without the necessity to use \( \sigma \). Substituting \( n = 7 \) and gives \( p = 0.094 \) or approximately 10 per cent.