Transmission Lines and Surges

A transmission line consists of any system of conductors to transmit electrical energy between two or more points.
Characteristic Impedance

The input impedance of an infinitely long line of constant cross-section is constant and is called the characteristic impedance of the line.

\[ Z_0 = \frac{V}{I} \]

The current \( I \) flows into the line to charge up the capacitance between the lines. It will flow as long as the voltage source is connected.
If a short length of line is cut off the input of an infinitely long line, the input impedance of the remaining infinitely long line will be the same characteristic impedance.

Similarly, if a short length of the line is terminated in an infinite length of the same line, it will behave as an infinite line and its input impedance will be the characteristic impedance.

The infinite line may be replaced by its characteristic impedance without disturbing the electrical conditions on the short line.

There can be no reflections from the end of the line (infinite or terminated in its characteristic impedance).
Transmission Line Equations

Consider a section of transmission line ($\delta z$), the line has a series resistance, $R - \Omega/m$, a series inductance $L - H/m$, a shunt conductance, $G - S/m$ and a shunt capacitance, $C - F/m$.

Kirchhoff’s current law:

$$I - (I + \delta I) = G\delta z V + C\delta z \frac{\partial V}{\partial t}$$
When $\delta z \to 0$, the equation reduces to

$$-\frac{\partial I}{\partial z} = GV + C \frac{\partial V}{\partial t}$$

Similarly, applying Kirchhoff's voltage law we have

$$-\frac{\partial V}{\partial z} = RI + L \frac{\partial I}{\partial t}$$

For low loss transmission lines $R=0$ and $G=0$

$$\frac{\partial I}{\partial z} = -C \frac{\partial V}{\partial t} \quad \text{and} \quad \frac{\partial V}{\partial z} = -L \frac{\partial I}{\partial t}$$

Eliminating either $I$ or $V$ and then differentiating with respect to $t$, we have
\[
\frac{\partial^2 V}{\partial z^2} = LC \frac{\partial^2 V}{\partial t^2} \quad \text{and} \quad \frac{\partial^2 I}{\partial z^2} = LC \frac{\partial^2 I}{\partial t^2}
\]

They are called transmission line equations. The solution for voltage is

\[
V = f(z + vt) + g(z - vt)
\]

where

\[
v^2 = 1 / LC
\]

and the related solution for current is

\[
I = \frac{1}{Z_0}[-f(z + vt) + g(z - vt)]
\]
where

$$Z_0 = \sqrt{\frac{L}{C}}$$

**Line Termination**

What happens if the line is terminated with other impedance, $Z_t$, rather than its characteristic impedance?

Two consequences: part of the signal transmits through the junction and the rest is reflected back.
The electrical signals at the junction are an incident wave, \( V_i \) and \( I_i \) where \( V_i = Z_0 I_i \) ⇒ a transmitted wave \( V_t \) and \( I_t \) where \( V_t = Z_t I_t \) and a reflected wave \( V_r \) and \( I_r \) where \( V_r = -Z_0 I_r \)

At the junction, the total current flowing into the junction is zero and the total voltage on each side of the junction is equal, i.e.

\[
I_i + I_r = I_t
\]

and

\[
V_i + V_r = V_t
\]
Bring the voltage and current relationship into current equation

\[ \frac{V_i}{Z_0} - \frac{V_r}{Z_0} = \frac{V_t}{Z_t} \]

\[ \Rightarrow V_i - V_r = \frac{Z_0}{Z_t} V_t \]

Combine with the voltage equation

\[ \frac{V_r}{V_i} = \frac{Z_t - Z_0}{Z_t + Z_0} \]

This is called the reflection coefficient and is donated by \( \rho \).

\[ \rho = \frac{\text{reflected voltage}}{\text{incident voltage}} = \frac{V_r}{V_i} = \frac{Z_t - Z_0}{Z_t + Z_0} \]
Similarly, the transmission coefficient $\tau$ can be defined:

$$\tau = \frac{\text{transmitted voltage}}{\text{incident voltage}} = \frac{V_t}{V_i} = \frac{2Z_t}{Z_t + Z_0}$$

Note: $\tau - \rho = 1$ and if $Z_t = Z_0$ then $\rho = 0$, $\tau = 1$

**Surges On The Lines**

Power transmission and distribution systems involve connecting transmission lines and equipment. A voltage surge travels along the line at the wave velocity of the line ($v^2 = LC$).
A junction between three transmission lines

The junction appears as a terminating impedance given by the characteristic impedance of the two lines in parallel,

\[ Z_j = \frac{Z_1 Z_2}{Z_1 + Z_2} \]
The reflection coefficient is given by

\[ \rho = \frac{Z_j - Z_0}{Z_j + Z_0} \]

The transmitted voltage appears across \( Z_1 \) and \( Z_2 \) in parallel.

Complicated situation occurs if any surge is reflected from two ends of the lines. The voltage at the junction will be time dependent. The situation is best described by a space-time diagram (also best known as the Bewley Lattice Diagram).
The Bewley Lattice Diagram:
The distance along the transmission line is plotted horizontally and time following the initial switching closure is plotted vertically.

The progression of any particular voltage step on the line is plotted as a line on the space-time diagram.

The size of each voltage step must be indicated for each line on the diagram.
Example 1: A source of 160 V, internal impedance 0.6 \( Z_0 \) is connected through a transmission line of characteristic impedance, \( Z_0 \), length \( l \) to a load of impedance 9 \( Z_0 \). Given that \( T \) is the time taken for the switching surge to make one traverse of the transmission line, plot the voltage at the end of the line and the voltage at the mid-point of the line until time 6\( T \) after the source is connected to the line.

Solution: The initial voltage on the line is given by

\[
V = \frac{Z_0}{1.6Z_0} \times 160 = 100V
\]
The source and load reflection coefficients are given by

\[ \rho_l = \frac{9Z_0 - Z_0}{9Z_0 + Z_0} = 0.8 \]

and

\[ \rho_s = \frac{0.6Z_0 - Z_0}{0.6Z_0 + Z_0} = -0.25 \]
The Bewley lattice diagram:
--- at end of the line

----- at mid-point of the line
Example 2: A long length of 132 kV line has a surge impedance of 300 Ω. A transformer of characteristic impedance 950 Ω is connected via a cable of surge impedance 50 Ω and travel time 3 μs to a point mid-way along the line.

(i) If a lightning surge of step function 750 kV magnitude travels along the line towards the line-cable junction estimate the voltage profile at the junction and the transformer over a 12 μs period following the arrival of the surge at the junction.

(ii) If a surge arrester with a protective level of 400 kV is installed at the line-cable junction estimate the voltage profile at the transformer.
Solution:

At junction line-cable (1)

The transmission and reflection coefficients for the surge from the left are

\[ \tau = \frac{2 \times 50 \times 300}{50 + 300} = 0.25 \quad \text{and} \quad \rho = \tau - 1 = -0.75 \]

The transmission and reflection coefficients for the wave reflected from the cable-transformer junction are

\[ \tau = \frac{2 \times 300 \times 300}{300 + 300} = 1.5 \quad \text{and} \quad \rho = \tau - 1 = 0.5 \]
At the junction cable-transformer (2)
The transmission and reflection coefficients for the wave from the line-cable junction are

\[ \tau = \frac{2 \times 950}{50 + 950} = 1.9 \quad \text{and} \quad \rho = \tau - 1 = 0.9 \]

i.e.
The Bewley lattice diagram:

750kV

- 187.5
- 168.75
- 84.83
- 76
- 38

- 3µs
- 9µs
- 12µs

6µs
The voltage profiles at the junction and transformer
Example 3: Line terminated with a transformer (taken as an L-C parallel combination):

A transformer is connected to the end of a transmission line with a surge impedance of Z. If a rectangular wave of \( e(t) \) travels through the line and strikes the transformer, find the surge voltage on the transformer.
Solution: \( e(t) = EU(t), \ Z_1 = Z \) and \( Z_2 = L \) and \( C \) in parallel

Using Laplace transform:

\[
e(s) = \frac{E}{s}, \ Z_1 = Z \quad \text{and} \quad Z_2 = \frac{Ls}{Cs} = \frac{s}{C} \left( \frac{1}{s^2 + \frac{1}{LC}} \right)
\]

Therefore, the reflection coefficient is

\[
\rho = \frac{Z_2 - Z_1}{Z_2 + Z_1} = -\frac{s^2 - \frac{s}{CZ} + \frac{1}{LC}}{s^2 + \frac{s}{CZ} + \frac{1}{LC}}
\]
Let $\alpha=1/CZ$ and $\omega_0^2 = 1/LC$ then

$$\rho = \frac{Z_2 - Z_1}{Z_2 + Z_1} = -\frac{s^2 - \alpha s + \omega_0^2}{s^2 + \alpha s + \omega_0^2}$$

The transmitted wave is

$$e'(s) = \tau \cdot e(s) = (1 + \rho)e(s) = (1 - \frac{s^2 - \alpha s + \omega_0^2}{s^2 + \alpha s + \omega_0^2}) \frac{E}{s}$$

$$= \frac{2\alpha E}{s^2 + \alpha s + \omega_0^2} = \frac{2\alpha E}{n-m} \left( \frac{1}{s+m} - \frac{1}{s+n} \right)$$

where $n$ and $m$ are roots of the denominator.
Taking the inverse transform:

(i) if $\omega_0^2 < (\alpha / 2)^2$ then we have real value for $n$ and $m$

$$e'(t) = \frac{2\alpha}{n-m}[\exp(-mt) - \exp(-nt)]EU(t)$$

(ii) if $\omega_0^2 > (\alpha / 2)^2$ then we have complex solution

$$e'(t) = \frac{2\alpha}{\omega_0^2 - (\alpha / 2)^2}\exp(-\alpha t / 2)\sin[\sqrt{\omega_0^2 - (\alpha / 2)^2} t)]EU(t)$$
Application - HV Pulse Generators

Pulse generator at high voltage often uses the Blumlein pulse forming line. Two equal lengths of transmission line are used together with a switch to construct the generator as shown in Figure 1. DC power supply charges up the two lines to V. The transmit time for each line is $\delta$.

Use the Bewley lattice diagram to determine the voltage profile up to $8\delta$ across the load $Z_l$ after switch is closed (assuming time $t=0$) for $Z_l=2Z_0$. 
Figure 1 Basic Blumlein pulse-forming line
The lattice diagram is drawn for the general case i.e. $Z_l \neq 2Z_0$.

After switch closure, which initiates pulse-forming action, the end of line 1 next to the switch is effectively shorted, thus the reflection coefficient at this end of line 1 is $-1$. At the open end of line the reflection coefficient is $+1$.

We assume that the load and line impedances are purely resistive. In this case the reflection coefficient at the load is given by

$$\rho = \frac{(Z_l + Z_0) - Z_0}{(Z_l + Z_0 + Z_0)} = \frac{Z_l}{Z_l + 2Z_0}$$

The reflected step at the load thus has an amplitude

$$V_\text{\textsc{r}} = -\rho V = -V \frac{Z_l}{Z_l + 2Z_0}$$
The step $V_T$ transmitted on to the load and second line has an amplitude given by

$$V_T = V_+ + V_- = -V (1 + \frac{Z_l}{Z_l + 2Z_0}) = -2V \frac{Z_l + Z_0}{Z_l + 2Z_0}$$

This step is divided among the load and second line, and therefore the fraction of the step that is impressed on the load is given by

$$V_l = -2V \frac{Z_l}{Z_l + Z_0} \cdot \frac{Z_l + Z_0}{Z_l + 2Z_0} = -2V \frac{Z_l}{Z_l + 2Z_0} = -\alpha V$$

where

$$\alpha = \frac{2Z_l}{Z_l + 2Z_0}$$
The fraction of the step propagated on to the second line $V_{2T}$ is given by

$$V_{2T} = \frac{Z_0}{Z_l + 2Z_0} (-2V) = -\beta V$$

where

$$\beta = \frac{2Z_0}{Z_l + 2Z_0}$$
The lattice diagram is shown below:
The potential on the load is determined by adding sequentially, the fraction of propagating steps which are impressed on the load to the initial potential impressed on the load \(-\alpha V\). Thus the potential on the load is given by

\[ V_l = -\alpha V[1 - (\rho + \beta) + (\rho^2 - \beta^2) - (\rho + \beta)(\rho^2 - \beta^2) + (\rho^2 - \beta^2)^2 + \cdots] \]

where each successive term in the square brackets is delayed by twice the transient time \(\delta\) of the two lines. It can be seen that

\[ \rho + \beta = 1 \]

Thus we have

\[ V_l = -\alpha V[1 - 1 + (\rho - \beta) - (\rho - \beta) + (\rho - \beta)^2 - (\rho - \beta)^2 + \cdots] \]
It is clear from the above equation that when $Z_1=2Z_0$, $\rho=\beta=0.5$ and $\alpha=1$. The voltage profile across the load over $8\delta$ is shown in figure.

\[ \delta = \frac{l \sqrt{\varepsilon_r}}{c} \]

$l$ is the length of the transmission line and $c$ is speed of light in vacuum.