TRANSMISSION LINES AND SWITCHING SURGES

The aim of this chapter is to introduce the reader to the effects of time delay and switching surges in transmission lines.

1 INTRODUCTION

A transmission line consists of any system of conductors that can be used to transmit electrical energy between two or more points. When a voltage source is connected to the input of a transmission line, the potential difference all along the line cannot rise instantaneously to that of the source. Time is needed for the transfer of energy corresponding to the potential difference between the conductors of the line. The special theory of relativity also indicates that an instantaneous change of potential along the whole length of the line is impossible. No electrical signal can be transmitted at a speed greater than the speed of light and time is needed for charge to travel along a transmission line.

A circuit connected by a transmission line of length $l$ is shown in Figure 1. Two ammeters are connected in the circuit, one at the source and one at the load. On closing the switch, a current will flow in the ammeter $A_1$ immediately but the current will only flow in $A_2$ some time later. The first flicker in the ammeter $A_2$ will occur at a time $l/c$ after the switch was closed, where $c$ is the speed of light. The steady-state conditions for the current flowing in $A_2$ occur sometime later than that.

Consider any two-wire transmission line of infinite length as shown in Figure 2. The only condition that needs to be specified is that the line maintains a constant cross-section throughout its length. If it is connected to a voltage source as shown in Figure 2, a current will flow into the line in order to charge up the capacitance between the wires. The current will continue to flow for as long as the voltage source is connected because the line is of infinite length and can never be completely charged. The current will always flow even if there is no leakage conductance between the wires of the line. The ratio

$$\frac{V}{I} = Z_0$$  \hspace{1cm} (1)

is a constant for any particular line and is called the \textit{characteristic impedance} of the line. The input impedance of any infinitely long line is its characteristic impedance. If a short length of line is cut off the input of an infinitely long line, the input
impedance of the remaining infinitely long line will be the same characteristic impedance. Similarly, if a short length of line is terminated in an infinite length of the same line, it will behave as an infinite line and its input impedance will be the characteristic impedance. The infinite line may be replaced by its characteristic impedance without disturbing the electrical conditions on the short line, so that a short line terminated in its characteristic impedance behaves like an infinite line. The equivalence is shown in Figure 3.

![Fig. 3 Showing equivalence between an infinite line and a short terminated in its characteristic impedance.](image)

It takes time for electrical charge to travel along a transmission line. For an infinite line, there can be no reflections from the end of the line because the signal never reaches the end of the line. Similarly there can be no reflections from the end of the line that is terminated in its characteristic impedance because such a line behaves as if it were a line of infinite length.

### 2 TRANSMISSION LINE EQUATIONS

Consider any two-wire transmission line. The wire of the line has a series resistance, \( R \, \Omega/m \), and a series inductance, \( L \, H/m \). There is also a shunt capacitance, \( C \, F/m \), and a shunt conductance, \( G \, S/m \), between the two wires of the line. In many practical lines, the shunt conductance is negligible but it is included here for completeness. The effect of a short length of line, \( \delta z \), is shown in Figure 4. The current and voltage on the line are also shown in Figure 4. Although it is obvious from the circuit diagram that the current and voltage decrease as \( z \) increases along the line, both are shown to increase with increase of \( z \) so that the signs are correct in the mathematics.
Applying Kirchhoff’s current law to the circuit shown in Figure 4 gives

\[ I - (I + \delta I) = G \delta \dot{V} + C \delta \frac{\partial V}{\partial t} \]

In the limit, as \( \delta z \to 0 \), this equation reduces to

\[ -\frac{\partial I}{\partial z} = GV + C \frac{\partial V}{\partial t} \tag{2} \]

Applying Kirchhoff’s law to the voltage drop across R and L and neglecting the second-order small terms in \( \delta I \delta z \) gives

\[ V - (V + \delta V) = R \delta I + L \delta \frac{\partial I}{\partial t} \]

Similarly, this equation reduces to

\[ -\frac{\partial V}{\partial z} = RI + L \frac{\partial I}{\partial t} \tag{3} \]

Since most transmission lines are low loss and the series resistance, R, and the shunt conductance, G, are very small, the assumption will be made that

\[ R=0 \quad \text{and} \quad G=0 \]

so simplifying eqns. (2) and (3). They become

\[ \frac{\partial I}{\partial z} = -C \frac{\partial V}{\partial t} \tag{4} \]

and

\[ \frac{\partial V}{\partial z} = -L \frac{\partial I}{\partial t} \tag{5} \]

The current I will be eliminated from these two equations. First differentiate eqn.(5) with respect to \( z \),

\[ \frac{\partial^2 V}{\partial z^2} = -L \frac{\partial^2 I}{\partial t \partial z} \]

and differentiate eqn.(4) with respect to \( t \),
\[ \frac{\partial^2 I}{\partial t \partial z} = -C \frac{\partial^2 V}{\partial t^2} \]

Substituting one into the other gives

\[ \frac{\partial^2 V}{\partial z^2} = LC \frac{\partial^2 I}{\partial t^2} \quad (6) \]

Eliminating the voltage \( V \) from eqns. (4) and (5) by a similar process, we have

\[ \frac{\partial^2 I}{\partial z^2} = LC \frac{\partial^2 V}{\partial t^2} \quad (7) \]

Equations (6) and (7) are wave equations and a possible solution has the form

\[ V = f(z \pm vt) \quad (8) \]

\( f \) is any arbitrary function which is usually a sinusoidal function or combination of exponential functions. Differentiating eqn. (8) gives

\[ \frac{\partial V}{\partial z} = f'(z \pm vt) \]

\[ \frac{\partial^2 V}{\partial z^2} = f''(z \pm vt) \]

\[ \frac{\partial V}{\partial t} = \pm vf'(z \pm vt) \]

\[ \frac{\partial^2 V}{\partial t^2} = v^2 f''(z \pm vt) \]

where the prime denotes differentiation with respect to the argument. Therefore

\[ v^2 \frac{\partial^2 V}{\partial z^2} = \frac{\partial^2 V}{\partial t^2} \quad (9) \]

By comparison with eqn. (6)

\[ v^2 = \frac{1}{LC} \quad (10) \]

and \( v \) is the velocity of the wave.
Equation (6) is a second-order partial differential equation whose solution contains two arbitrary functions. Therefore the most general solution eqn. (6) is

\[ V = f(z + vt) + g(z - vt) \]  

(11)

where \( f \) and \( g \) are arbitrary functions whose shape is determined by boundary conditions in \( z \) and \( t \). Consider just one of these solutions,

\[ V = g(z - vt) \]

As shown by Figure 5, \( g(z - vt) \) is a wave of fixed shape travelling in positive \( z \) direction with a velocity \( v \). Similarly, \( f(z + vt) \) is a wave of shape travelling in the negative \( z \) direction with a velocity \( v \). The voltage on the line is given by the sum of these two waves.

![Fig. 5 An arbitrary wave \( g(z) \) travelling in the positive \( z \)-direction with a velocity of \( v \): (a) \( t=0 \), (b) \( t=t_1 \).](image)

The electric current flowing in the line is derived by substituting (11) into eqn. (4). Therefore

\[ \frac{\partial I}{\partial z} = -C \frac{\partial V}{\partial t} = -C[vf'(z + vt) - vg(z - vt)] \]

Integrating with respect to \( z \) gives

\[ I = Cv[-f(z + vt) + g(z - vt)] \]

and substituting from eqn. (10) gives

\[ I = \sqrt{\frac{C}{L}}[-f(z + vt) + g(z - vt)] \]

(12)

The constant \( \sqrt{L/C} \) has the dimensions of impedance and is called the characteristic impedance of the line. It gives a numerical ratio between the current wave and the
voltage wave on the line and is the same as the characteristic impedance of the infinitely long line discussed in Section 1. Therefore

\[ Z_0 = \sqrt{\frac{L}{C}} \]  

(13)

For the forward wave, \( V = g(z - vt) \),

\[ V = Z_0 I \]

and the current wave is exactly the same shape as the voltage wave. For the reverse wave, \( V = f(z + vt) \) and

\[ V = -Z_0 I \]

Again the current due to the reverse wave is a wave of exactly the same shape as the voltage wave but it is now flowing in the negative z direction, hence giving rise to the negative sign in the above equation. For both the forward and reverse waves, the current flows in the direction of travel of the wave.

3 LINE TERMINATION

So far we have considered only an infinite line and a line terminated in its characteristic impedance. Any other termination causes a signal to be reflected from the junction or the termination, which gives rise to backward waves on the line. Consider a junction between two different lines as shown in Figure 6.

\[ Z_0 \]
\[ Z_t \]
\[ V_r \]
\[ V_i \]
\[ V_t \]

Fig.6 A line of characteristic impedance of \( Z_0 \) connected to a line of characteristic impedance of \( Z_t \).

This is a transmission line of characteristic impedance \( Z_0 \) terminated by another line of characteristic impedance \( Z_t \). If we take the forward current flow as positive, the electrical signals at the junction are

an incident wave, \( V_i \) and \( I_i \) where \( V_i = Z_0 I_i \)

which gives rise to

a transmitted wave \( V_t \) and \( I_t \) where \( V_t = Z_I I_t \)

and
a reflected wave \( V_r \) and \( I_r \) where \( V_r = -Z_0 I_r \)

and the negative sign makes allowance for the fact that the current of reflected wave is flowing in the direction of travel of that wave.

At the junction, the total current flowing into the junction is zero and total voltage on each side of the junction is equal. Therefore

\[ I_i + I_r = I_t \quad (14) \]

and

\[ V_i + V_r = V_t \quad (15) \]

Substituting the voltages into eqn. (14) gives

\[ \frac{V_i}{Z_0} - \frac{V_r}{Z_0} = \frac{V_t}{Z_t} \]

which reduces to

\[ V_i - V_r = \frac{Z_0}{Z_t} V_t \quad (16) \]

Equations (15) and (16) make a pair of simultaneous equations which may be solved for \( V_i \) and \( V_r \), giving

\[ 2V_i = V_t (1 + \frac{Z_0}{Z_t}) \quad (17) \]

\[ 2V_r = V_t (1 - \frac{Z_0}{Z_t}) \quad (18) \]

Taking the ratio of eqns. (18) and (17) gives

\[ \frac{V_r}{V_i} = \frac{Z_t - Z_0}{Z_t + Z_0} \quad (19) \]

The ratio in eqn. (19) is called the reflection coefficient and is denoted by the symbol \( \rho \).

\[ \rho = \frac{\text{reflected voltage}}{\text{incident voltage}} = \frac{V_r}{V_i} = \frac{Z_t - Z_0}{Z_t + Z_0} \quad (20) \]

The current reflection coefficient is defined similarly and is obtained by substitution into eqn. (19),

\[ \rho_c = \frac{I_r}{I_i} = -\rho = \frac{Z_0 - Z_t}{Z_t + Z_0} \quad (21) \]
The negative sign occurs because the current in the reflected wave is flowing in the opposite direction to the current in the incident wave. From eqn. (17) we obtain

\[ V_t = \left( \frac{2Z_t}{Z_t + Z_0} \right) V_i \quad (22) \]

Therefore

\[ I_t = \left( \frac{2}{Z_t + Z_0} \right) V_i \quad (23) \]

The ratio given in eqn. (22) is defined similarly to the reflection coefficient to be the transmission coefficient,

\[ \tau = \frac{\text{transmitted voltage}}{\text{incident voltage}} = \frac{V_t}{V_i} = \frac{2Z_0}{Z_t + Z_0} \quad (24) \]

It has already been shown in Section 1 that a line may be replaced by its characteristic impedance. Therefore the terminating line in Figure 6 may be replaced by its characteristic impedance as shown in Figure 7. Then we have the situation of a line of characteristic impedance \( Z_0 \) terminated by an impedance \( Z_t \). As far as the incident wave is concerned, it does not whether the line is terminated by another line or by an impedance of same value. So eqns. (20) and (21) also apply to any line terminated in impedance other than its characteristic impedance. The transmitted portion of the wave is absorbed in the terminating impedance.

### 4 VOLTAGE STEP

When a steady voltage is switched onto a line, that voltage does not appear everywhere on that line instantaneously. A voltage surge travels along the line at the wave velocity of the line. At the same time, a steady current flows into the line in order to charge up the capacitance of the line. Consider a line connected through its characteristic impedance to a battery as shown in Figure 8 with its opposite end open circuited. When the switch is closed, the battery sees the characteristic impedance of the line in series with the source resistance \( R_s \). If \( R_s = Z_0 \), \( 1/2V \) appears as the voltage on the line and a voltage and current surge travels along the line at the wave velocity of the line as shown by Figure 8(b). The voltage and current surges travel together and before they arrive the line does not know that the switch has been closed. When the surge reaches the end of the line, a reflected surge is generated which is governed by the reflection coefficient and transmission coefficient at the end of the line. For an open circuit, \( Z_t = \infty \). Therefore

\[ \rho = 1 \quad V_r = V_i \quad I_r = -I_i \quad V_t = 2V_i \quad I_t = 0 \]
Where notation is taken from Figure 6. The reflected voltage and current surge is shown in Figure 8(c). It is seen that the reflected voltage adds to the voltage already on the line to give a voltage of \( V \) at the termination. The reflected current surge is negative, so that there is no current flowing into the termination. The final situation at the termination is that of a simple d.c. connection to an open circuit.

The situation shown in Figure 8(c) is at some time later than the situation shown in Figure 8(b). It takes a small, but finite, time for the switching surge to travel along the line and for the reflected surge to travel back. All that time, current is flowing into the line from the battery to charge up the capacitance of the line. During the forward travelling voltage step shown in Figure 8(b) it is easy to see that the current flows at a rate given by the product of voltage, capacitance per meter and velocity. During the passage of the reflected voltage step shown in Figure 8(c), charge is being accumulated at the same rate and the same current continues to flow until that voltage step has passed. At any point on the line, the voltage and the current remain constant until the voltage step arrives. When the surge returns to the source, the source appears to have the characteristic impedance of the line and no further reflections occur. The switching surge is over and the static d.c. conditions prevail on the line.

\[ V \]
\[ R_s \]
\[ Z_0 \]

(a)

(b)

(c)

Figure 8 Voltage and current surge on an open-circuited line: (a) circuit; (b) before the surge reaches the end of the line; (c) the reflected surge before it reaches the source.
The similar conditions for a line terminated in a short circuit are shown in Figure 9. The effect of the initial switching surge as shown in Figure 2.9(b) is the same as that shown in Figure 8(b) for the open-circuited line. However, the terminating conditions are different. They are $Z_t = 0$. Therefore

\[ \rho = -1 \quad V_r = -V_i \quad I_r = I_i \quad V_i = 0 \quad I_r = 2I_i \]

where the notation is taken from Figure 6. For this situation, the reflected voltage and current surge shown in Figure 9(c) is the dual of that for the open-circuited line as shown in Figure 8(c). The final situation is that of a simple d.c. connection to a short circuit.

Figure 9 Voltage and current surge on a short-circuited line: (a) circuit; (b) before the surge reaches the end of the line; (c) the reflected surge before it reaches the source.
If the line is terminated in its characteristic impedance, there is no reflected surge, and the final current and voltage on the line are the same as those shown in Figure 8(b) and Figure 9(b) even after the voltage step has reached the end of the line.

In the circuit described by Figures 8 and 9, the source impedance was assumed to be the same as the characteristic impedance of the line. If it is not, then the voltage is divided between the source impedance and the line in proportion to their impedances. Consider the circuit shown in Figure 10(a). As soon as the switch is closed, the circuit behaves as the circuit shown in Figure 10(b). The voltage $V$ is developed across the line which is given by

$$V = \frac{Z_0}{Z_0 + R_s} V_0 \quad (25)$$

If the line is not infinitely long or terminated in its characteristic impedance, any reflected voltage step sees the line as terminated in the source impedance, $R_s$. As far as the source termination is concerned, the voltage source is replaced by its Thevenin equivalent impedance, and there will be a source reflection coefficient given by

$$\rho = \frac{R_s - Z_0}{R_s + Z_0} \quad (26)$$

![Figure 10](image)

**Figure 10** The effect of the source impedance on the initial voltage on the line (a) circuit; (b) equivalent circuit at the instant after the switch is closed.

If a line is connected to two other lines in a junction as shown in Figure 11(a), the junction appears as a terminating impedance given by the characteristic impedance of the two lines in parallel,

$$Z_j = \frac{Z_{02}Z_{03}}{Z_{02} + Z_{03}} \quad (27)$$
The reflection coefficient is given by

$$\rho = \frac{Z_j - Z_{01}}{Z_j + Z_{01}}$$  \hspace{1cm} (28)$$

The transmitted voltage appears across the $Z_{02}$ and $Z_{03}$ in parallel. If an impedance is added in shunt across a line as shown in Figure 11(b), the calculation is the same as for the junction between three lines. The effective impedance at the point A is given by

$$Z = \frac{Z_{1}Z_{0}}{Z_{1} + Z_{0}}$$

and the reflection coefficient is given by

$$\rho = \frac{Z - Z_{0}}{Z + Z_{0}} = \frac{-Z_{0}}{2Z_{1} + Z_{0}}$$  \hspace{1cm} (29)$$

If any switching surge is reflected from both ends of the line, the situation is much more complicated than the situation described in Figures 8 and 9. It is best described by a space-time diagram (also called the Bewley lattice diagram). Distance along the transmission line is plotted horizontally across a page, and time following the initial switch closure is plotted vertically on the same page. The progression of any particular voltage step on the line is plotted as a line on the space-time diagram. The size of each voltage step must be indicated for each line on the diagram. The use of the space-time diagram is best demonstrated by a simple example.

Figure 11 (a) A junction between three transmission lines; (b) an impedance in shunt across a transmission line.
Consider a HV dc source of 180 kV, internal impedance $0.8Z_0$, connected to a load impedance of $4.0Z_0$ through a line of characteristic impedance $Z_0$ of length $l$, as shown in Figure 12(a). When the switch is closed, the switching step will travel along the line at the wave velocity of the line, $v$. The initial voltage step on the line is given by eqn. (25),

$$V = \frac{Z_0}{Z_0 + 0.8Z_0} \times 180kV = 100kV$$

The time of propagation of the voltage step along the line is $T = l/v$. When the voltage step reaches the end of the line, it is reflected with a reflection coefficient given by

$$\rho_r = \frac{4Z_0 - Z_0}{4Z_0 + Z_0} = 0.6$$

As the reflection coefficient is positive, the reflected voltage step increases the voltage on the line. When the reflected voltage step reaches the source, it is further reflected with a reflection coefficient given by

$$\rho_s = \frac{0.8Z_0 - Z_0}{0.8Z_0 + Z_0} = -0.111$$

As this time the reflection coefficient is negative, the reflected voltage now decreases the total voltage on the line. As the voltage step is reflected from each end of the line in turn, it gets smaller and the voltage on the line approaches the static value of 150 V. The space-time diagram is shown in Figure 12(b). It is seen that the voltage on the line reaches within 1 V of the static value after $4T$ and within 0.1 V of the static value after $6T$. For most circuits the effects of these switching surges may safely be ignored. However, in long lines or on high-speed computers, switching delays may be significant and in high-voltage systems allowance must be made for the highest voltage reached during switching.
Figure 12 Switching surges on a transmission line.
5 PROTECTION OF TRANSMISSION LINES FROM SURGES

There are different ways of protecting the lines from surges.

**Ground wires on transmission lines (protecting from lightning surges)**

Typically, the 10-150kA return stroke of a lightning bolt releases some 1,000 to 10,000 MJ of energy. Clearly, if this 20 kA return stroke current is attached to a line, it would split in the two directions from the point of strike into 10 kA each. In the immediate aftermath of the hit, no backward wave would exist. Forward waves would be created travelling in opposite directions. (One could argue that these two waves are the forward and backward waves since they travel in opposite directions). As such for a typical overhead line of characteristic impedance 400 \( \Omega \),

\[
V = Z_o I = 400 \times 10 \times 10^3 = 4MV
\]

Naturally, a voltage surge of 4 MV has to be avoided! This is done by carrying a ground wire on top of the transmission line as shown in Figure 13.

![Ground wires on transmission lines](image)

Figure 13 A three-phase transmission system with earth wires on top.

Shielding theory is simply based on the fact that the object that is closer to the cloud sets up a higher electric field and thereby is more prone to be struck. The protection region depends on several factors. Most important factor is the shielding angle \( \theta \) as shown in Figure 13. A typical value for \( \theta \) is 48\(^\circ\).

**Surge Diverters—the Rod Gap**

When surges as from lightning and switching travel down a line, they can cause havoc. For an example, Figure 14 shows a line entering a transformer and carrying a surge. Should the surge enter the transformer, the high voltage of the surge can stress the insulating materials such as pressboard and insulators and make them breakdown. A rod gap is a surge diverter. It is simply two rods, a designed distance apart, and placed between the line and ground as shown in Figure 14. The gap is such that when the normal operating line voltage is applied across the gap, nothing happens. That is,
under normal operation, it presence is not felt. But when a large surge arrives on the line, the voltage of the surge breaks down the air between the rod tips—the gap—and the resulting spark conducts current to the ground, thereby dissipating the energy of the surge by diversion and sparing the transformer.

![Surge protection of a transformer using a surge diverter.](image)

**Surge absorbers**
Surge absorbers, unlike surge diverters, are connected in series with the line, as shown in Figure 15 where the absorber is a box of lead dioxide (PbO₂) pellets. Lead dioxide is a good conductor and carries the load under normal operation. But when a surge arrives, the large current associated with the surge heats the pellets, which then turn to lead oxide (PbO), a good insulator, and thereby limit the current. The disadvantage is that the lead dioxide needs to be replaced once it turns to lead oxide.

![Surge protection of a transformer using a surge absorber.](image)

**Non-linear resistor Characteristics and requirements**
Non-linear resistor can be used in place of the rod gap. The old type of surge diverters is usually made of silicon carbide and has a voltage/current relationship given by

\[ V = kI^\beta \]

where \( k = 1300 \) and \( \beta = 0.2 \).
The surge diverter can pass a current of 5 kA at only three times the system line voltage and yet only pass 4-5 A at the crest value of normal system voltage. In recent years, however, development of zinc oxide, as a non-linear resistance element, has taken place so rapidly and been so successful, that this material is now replacing SiC as the non-linear resistance element in surge diverters. The zinc oxide (ZnO) non-linear resistor is a sintered element, but is produced from a mixture of zinc oxide and other metallic oxides in power form, pressed and heated to a high temperature to produce the final element.

**Operating Principle**

The arrester consists of the main component of ZnO and several kinds of additives, which are mixed, granulated, formed and sintered into a complete block with electrodes on the both surfaces. The picture (figure 17) shows a typical internal view of the element under a scanning electron microscope. Composed of series and parallel connections of the ZnO grains and the boundary layers, the energy band model is shown. The boundary layer has a very high impedance for a small current region, so the normal operating voltage is almost all applied to its boundary layer.

![Energy Band Model](image)

**Figure 16** V-I characteristic of SiC surge diverter.

![V-I Characteristic](image)
Figure 18 below shows the V-I characteristics of the 550kV class surge arrester for example. The current flowing through the element at the nominal line-to-ground voltage is on the order of micro-ampere. As a result, the protective performance of the elements is stable in long term use, even though without series gaps.