Why study fluids?

Fluids and their behaviour are central to many areas of Engineering:

- Large scale civil engineering – dams, bridges etc.
- Naval (maritime) engineering – ship design, structural design of rigs
- Environmental engineering – river management, experimental apparatus

These are largely design areas where an understanding of the behaviour of fluids is central to optimising efficient systems and objects. We also have, for example:

- Aeronautical engineering – aircraft design

where the fluid is an integral part of how the machine works, with the design using the fluid (air) to produce a mechanical effect (lift).
One of the most interesting uses of fluids in engineering is as a means of transmitting energy from one location to another, fluid power systems, e.g.:

- Internal energy transfer - thermodynamics.
- Mechanical: hydraulics, pneumatics etc.

**Pumped storage hydroelectric**

- Racoon Mountain, Tennessee
  - [http://www.tva.gov/sites/raccoon-mt.htm](http://www.tva.gov/sites/raccoon-mt.htm)
- Ben Cruachan, Loch Awe, Scotland
  - [http://www.visitcruachan.co.uk/](http://www.visitcruachan.co.uk/)

**The Properties of Fluids**
There are generally three states of matter: solid, liquid and gas. The latter two are different from the first:

- A solid has a permanent resistance to deformation from pressure and stress in that as the deformation continues the restoring action increases. In addition, as long as the material structure is not damaged by exceeding the elastic limit, the material will return to its original shape once the deforming force is removed.
- A liquid or a gas will flow under the action of the deformation and continue to do so as long as the applied force/pressure continues. Neither of these materials will return to their original shape once the deforming force is removed.

This gives the definition of a fluid: A fluid is something which deforms continuously under the action of shearing forces.

We will discuss the nature of shearing forces later.

The differences between the different states of matter are largely but not completely a result of the amount of energy that the individual molecules which comprise the material possesses (Douglas Ch1). The molecules are always in motion but behave differently depending on local electrostatic and quantum mechanical interactions (Van der Waals forces and chemical bonding).

As will be discussed later in this section and again in the sections on thermodynamics, the temperature is related to the internal energy of the material and the kinetic energy of the molecules. As the temperature rises, the molecules gain more kinetic energy and the movement and forces of impact from collisions increase, resulting in an increase in spacing.

In a solid, the molecules are closely packed and cannot move due to a combination of physical restraint due to nearest neighbours, attractive short range electrostatic forces or shared electron bonding.

In a liquid, the spacing is normally slightly greater (but not always – ice forms from hydrogen bonding with a greater spacing than water). The molecules possess sufficient energy to slide across one another and to displace neighbours. The intermolecular forces are still sufficient to hold the material together. A liquid is generally incompressible due to this nature.

A liquid will fill a container, wetting the base and the sides. The upper surface is a fixed pressure surface related to the interaction of pressure and gravity (more later). Emptying the container requires mechanical force (work).
Liquid or gas?

In a gas, the spacing is generally an order of magnitude greater than for a liquid. The molecules possess sufficient energy to render the intermolecular forces weak and the material has little or no structure. The molecules travel large distances (large mean free path) and interact with each other largely in the form of collisions.

A gas expands to fill a container, exerting a force (pressure on all internal surfaces). Removing the container allows the gas to expand outwards and emptying the container can be achieved by opening a hole.

Continuity

The continuum approximation allows simple models to be constructed in terms of material properties. Fluids are comprised of molecules, which slip over and around each other but in the continuum approximation they are considered to be continuous objects. Atomistic calculations (based on statistical mechanics) are impossible for large scale systems; one atom is \( \sim 10^{-10} \text{m} \) (angstrom \( \AA \)).

The continuum approximation is determined by a scaling argument referred to as the Knudsen number:

- If the Knudsen number is near or greater than one, the mean free path of a molecule is comparable to a length scale of the problem, and the continuum assumption as in fluid mechanics is no longer a good approximation. Statistical mechanics must then be used.
- If it is much less than one, the continuum mechanics formulation of fluid mechanics should be used.

The Knudsen number is defined as:

\[
Kn = \frac{\lambda}{L} = \frac{k_B T}{\sqrt{2\pi m P L}}
\]

- \( \lambda \) — mean free path (m)
- \( L \) — typical physical length scale (m)
- \( k_B \) — Boltzmann’s constant
- \( T \) — temperature (K)
- \( m \) — particle diameter (m)
- \( P \) — total pressure (Pa)

For water

\[
\lambda = \frac{1.38 \times 10^{-23} \text{J}}{8 \times 10^{-7} \text{m}^2} = 1.64 \times 10^{-10} \text{m}
\]

\[
Kn = \frac{\lambda}{L} \ll 1 \Rightarrow L \gg 10^{-10} \text{m}
\]

Water vapour

\[
\lambda = \frac{1.38 \times 10^{-23} \text{J}}{8 \times 10^{-7} \text{m}^2} = 2.6 \times 10^{-7} \text{m}
\]

\[
Kn = \frac{\lambda}{L} \ll 1 \Rightarrow L \gg 2.6 \times 10^{-7} \text{m}
\]
Incompressibility

Compressible fluid: pressure variations large enough to cause significant changes in density. If pressure variations are much smaller than those required to cause bubbles (cavitation), or flows with speeds much smaller than speed of sound are termed incompressible. For acoustic wave problems rather than surface waves, the fluid must be compressible.

For compressible or incompressible fluid dynamics, the Mach number of the problem can be used. If the Mach number is much smaller than one, then the fluid can be considered to be incompressible. This is normally used in gas flow dynamics.

\[ M = \frac{V}{u_c} \]

Alternatively, incompressibility for a liquid can be considered by examining the change in volume arising from a change in pressure. The usual quantity used to measure this is the bulk modulus of elasticity \( E \):

\[ E = \frac{-\Delta P}{\Delta V/V_0} \]

<table>
<thead>
<tr>
<th>liquid</th>
<th>Bulk modulus (10^6 Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>water</td>
<td>2,179</td>
</tr>
<tr>
<td>ethanol</td>
<td>896</td>
</tr>
<tr>
<td>oil</td>
<td>1,300</td>
</tr>
<tr>
<td>mercury</td>
<td>24,790</td>
</tr>
</tbody>
</table>

For most liquids, changes in pressure \(-10^6\)Pa do not produce significant compression.

Expanding on this, in the limit as \( \Delta P \to 0 \)

\[ E = -V \frac{dp}{dV} \]

If we take the equation for the unit mass

\[ V = \frac{1}{p} \]

and differentiate

\[ VdP + pdV = 0 \]

\[ dV = \left( \frac{E}{p} \right) dp \]

Substituting gives

\[ E = -p \left( \frac{dp}{dV} \right) \]
Incompressibility

This shows that there is a relationship between pressure and density and therefore between pressure and temperature. Compressibility therefore also depends on the changes in temperature during compression. This is the basis of much of what will be discussed later in the sections on thermodynamics.

The bulk modulus for gases is not constant (proportional to pressure) and there are different conditions: isothermal and adiabatic which affect the value.

Fluid properties

There are several fundamental properties that are important in fluid mechanics. Most are specific i.e. per unit volume.

Mass density: mass per unit volume. Density at a point is the limiting mass of a small volume –

\[ \rho = \lim_{V \to 0} \frac{\delta m}{\delta V} \]

\( V \) is the limiting volume, which is small but cannot be zero and still hold the continuum approximation: \( V \) must be large compared to the mean free path (liquid) or the mean distance between (gas) the molecules.

Units - kg m\(^{-3}\)

Typical values

- water \( \rho = 1000 \text{ kg m}^{-3} \)
- air \( \rho = 1.23 \text{ kg m}^{-3} \)

Fluid properties

Specific weight \( \nu \):

Defined as weight per unit volume, depending on the local value of the gravitational attraction \( g \). Derived from Newton’s second law:

\[ \nu = \rho g \]

Units - N m\(^{-3}\)

Typical values

- water \( \nu = 9.81 \times 10^3 \text{ N m}^{-3} \)
- air \( \nu = 12.07 \text{ N m}^{-3} \)
Fluid properties

Relative density or specific gravity $\sigma$:

defined as the ratio of the mass density to a reference standard (water at 4°C at atmospheric pressure (sea level)).

$$\sigma = \frac{\rho}{\rho_{\text{water at 4°C}}}$$

Unitless

Typical values

- Water $\sigma = 1$
- Oil $\sigma = 0.9$

In addition, there is an occasionally used term specific density which is the reciprocal of the mass density or average volume per unit mass.

Fluid properties

Temperature will be discussed in more detail later. Important to note here is that scaling values and properties requires the use of the absolute temperature scale in Kelvin.

Shearing and viscosity

Fluids flow continuously under the action of shearing forces which cause deformation.

If the fluid is at rest (hydrostatics), there are no shear forces acting on the fluid. All forces act perpendicular to their planes of action. When the fluid is moving, shear stresses are produced between the different parts of the fluid as they move past each other with different velocities. If the velocity of the fluid is uniform, then shear stress is again zero.
Shearing and viscosity

In fluid devices and systems, we are interested in the motion of the fluid past objects and across surfaces. The fluid at the boundary with the solid object is "stuck" and has zero velocity relative to the surface.

As distance from the surface increases, the velocity of the fluid increases.

Shearing and viscosity

Assume we have a fluid element with depth \( d \) and length \( L \). This is a section of the moving fluid and has shearing forces acting on the top surface and the bottom surface.

The shearing force \( F \) acts on an area equal \( A = L \times d \).

The force per unit area is the shear stress \( \tau = F/A \).
Shearing and viscosity

The deformation is measured by the angle \( \phi \), the shear strain. The shear strain is proportional to the shear stress and in a solid it is a fixed value for a given value of the stress. In a fluid, the shear strain increases continually with time as the fluid flows.

Over a time \( t \), the point \( P \) travels a distance \( \Delta x \).

From the figure, the shear strain is \( \phi = \Delta x / \Delta y \).

The rate of shear strain (shear strain per unit time) is

\[
\frac{\Delta \phi}{\Delta t} = \frac{\Delta x}{\Delta y} \frac{\Delta u}{\Delta y}
\]

where \( \Delta u \) is the velocity of \( P \).

Shearing and viscosity

Experimentally, for many fluids, the rate of shear strain is proportional to the shear stress so that

\[
\tau = \text{constant} \times \frac{\Delta u}{\Delta y}
\]

The velocity is written as \( \Delta u \) since it is a relative velocity, i.e. the velocity arising from the movement of \( P \) relative to a stationary base of the fluid volume. As the distance \( \Delta y \to 0 \) the equation for the shear stress becomes \( \tau = \mu \frac{\Delta u}{\Delta y} \), with the shear stress proportional to the gradient of the velocity in the transverse direction.

This is Newton’s definition of viscosity (law of viscosity) and the definition of the dynamic viscosity \( \mu \).

Dynamic viscosity

Units:

\[
\mu = \frac{\eta}{dy} \text{ N m} \text{ m}^{-1} \text{ s} \text{ Pa.s}
\]

\[
\eta = \frac{\mu}{g} \text{ kg m} \text{ s} \text{ Pa.s}
\]

Additional unit: poise = g / cm.s = 0.1 Pa.s
centipoise = 0.001 Pa.s = 1 mPa.s

Typical Values:

<table>
<thead>
<tr>
<th>Fluid</th>
<th>cPoise</th>
<th>Fluid</th>
<th>cPoise</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>0.018</td>
<td>Crude oil (35°C)</td>
<td>1300-5000</td>
</tr>
<tr>
<td>Acetone</td>
<td>0.3</td>
<td>Honey</td>
<td>3,000-10,000</td>
</tr>
<tr>
<td>Water (20°C)</td>
<td>1.02</td>
<td>Ketchup</td>
<td>50,000-70,000</td>
</tr>
<tr>
<td>Ether</td>
<td>1.2</td>
<td>Bitumen</td>
<td>1-10³</td>
</tr>
<tr>
<td>Mercury</td>
<td>1.55</td>
<td>Pulpit</td>
<td>1-10⁵</td>
</tr>
<tr>
<td>Olive oil</td>
<td>94</td>
<td>Tar pitch</td>
<td>3-10¹⁴</td>
</tr>
<tr>
<td>Machine oil</td>
<td>102</td>
<td>Soda glass</td>
<td>1-10²³</td>
</tr>
</tbody>
</table>
Kinematic viscosity

Many calculations in fluid mechanics use the kinematic viscosity:

\[ \nu = \frac{\mu}{\rho} \]

Since both dynamic viscosity and density are properties of the material, then so is the kinematic viscosity.

Units:

- \( \nu = \frac{\mu}{\rho} \) m/s kg m\(^3\)/kg s
- Additional unit: stoke = cm\(^2\)/s = 10\(^{-4}\) m\(^2\)/s
- centistoke = 10\(^{6}\) m\(^2\)/s = 1 mm\(^2\)/s

Typical values:
- water: 1.14 centistoke (1.14\times10\(^{-6}\) m\(^2\)/s)
- air: 14.6 centistoke (1.46\times10\(^{-5}\) m\(^2\)/s)

Newtonian and non-Newtonian fluids

Fluids are generally classified as Newtonian (ones which obey Newton’s law of viscosity) and non-Newtonian, those that more generally obey:

\[ \tau = A + B \left( \frac{d\omega}{d\phi} \right) \]

Non-Newtonian fluids

- Plastic – shear stress must reach a minimum threshold value before flow commences. Shear stress subsequently increases with shear rate with A, B and \( \eta \) constant.
- Pseudo-plastic – dynamic viscosity decreases as rate of shear increases, e.g., colloidal solutions, milk, cement.
- Thixotropic – dynamic viscosity decreases with time when shear applied.
- Dilatant – dynamic viscosity increases as the rate of shear increases.
- Rheopectic – dynamic viscosity increases with time when shear applied.
- Viscoelastic – behave as Newtonian fluids in the steady state but behave as plastics under sudden changes in shear stress.
Causes of viscosity

In a gas, the molecules are constantly moving and attractive forces between the molecules are weak. Viscosity arises from the interchange of molecules from one layer to another. As slower moving molecules travel upwards due to Brownian motion, they enter the faster moving layer above, producing a drag force due to collisions, acting on the faster molecules to slow them down. Likewise, as the faster moving molecules diffuse downwards into the slower layer, the slow moving molecules experience a drag force acting to increase their velocity.

In a fluid, viscosity arises both from molecular exchange and the attractive intermolecular forces. As a result, temperature has a large effect on the viscosity of the fluid.

Viscosity versus temperature

Surface tension

Although molecules are constantly moving in a liquid, the attractive forces between the molecules are strong and unidirectional.

At the surface with another fluid, for example air, the attractive forces across the interface are unbalanced, with the liquid molecules tending to be pulled inwards.

This makes the interface behave like an elastic membrane under tension, which acts to reduce the free surface area.

Surface tension is normally denoted \( \sigma \).
In a tube lowered into a fluid, the surface tension acts to ‘wet’ the inside surface.

Depending on the surface tension, the liquid will rise up the tube or be depressed below the surface.

- **Vertical Force** = upward component of surface tension × perimeter
  \[ \sigma \cos \theta \times \pi d \]

In steady-state, weight raised = \( \rho g \left( \frac{\pi}{4} \right) d^2 h \)

**Equating:**

\[ \sigma \cos \theta \times \pi d = \rho g \left( \frac{\pi}{4} \right) d^2 h \]

**Gives:**

\[ h = \frac{4\sigma \cos \theta}{\rho g d} \]