ELEC3215
FLUIDS AND
MECHANICAL MATERIALS

Fluids and Thermodynamics:
The Energy Equation
The Energy equation

We previously mentioned the energy equation in the Bernouilli form. In this section, we will examine the derivation of the energy equation for compressible fluids and discuss its relevance to Thermodynamic systems. We will then return to some fluid mechanics problems and examine the applications that the energy equation is put to in determining losses.

The energy equation is a general summary of the energy contained in and transferred through generic open systems. Determining the behaviour of the fluid and the system requires an understanding of the different types of energy and the relationship with the mechanical nature of the fluid. In thermodynamics, the application of the energy equation relates the heat transfer and work done components to the action of the fluid on it’s surroundings.
1\textsuperscript{st} Law of Thermodynamics in Open Systems

**Open system:** In an open system both mass and energy can cross the boundary.

Example: a gas turbine system:
Open system model

\[ dm \]

\[ Z_1 \]

\[ Z_2 \]

\[ Q \]

\[ W \]
Open systems: mass and energy

Assumption: the energy of the system remains constant, i.e. steady flow. For single/entry/exit system the mass flow rate expressed in terms of continuity equation is:

\[ \dot{m} = \rho_1 A_1 C_1 = \rho_2 A_2 C_2 \]

where \( \rho \) is the fluid density, \( A \) is the duct cross-sectional area and \( C \) is the mean fluid velocity.

The energy flowing into and out of the system consists of internal energy of the fluid, kinetic energy and potential energy.

Energy in: \((u_1 + \frac{C_1^2}{2} + gZ_1)dm\)  Energy out: \((u_2 + \frac{C_2^2}{2} + gZ_2)dm\)
Open systems: mass and energy

The net heat flow is \( Q \).

The work in this case consists of two components: external work \( W \) (shaft work) and internal work (flow work).

Flow work is \( p_1v_1 \) for the fluid entering and \( p_2v_2 \) for the fluid leaving.

Applying the 1st law (energy conservation) to the system

\[
(u_1 + \frac{C^2}{2} + gZ_1)dm + p_1v_1dm + Q = (u_2 + \frac{C^2}{2} + gZ_2)dm + p_2v_2dm + W
\]
Rearranging and dividing by \( dm \) gives

\[
\frac{Q}{dm} - \frac{W}{dm} = (u_2 + p_2 v_2) - (u_1 + p_1 v_1) + \frac{1}{2}(C_2^2 - C_1^2) + g(Z_2 - Z_1)
\]

Noting that enthalpy \( h = u + pv \), the 1st law for an open system is the steady flow energy equation (SFEE) given by

\[
qu - w = (h_2 - h_1) + \frac{1}{2}(C_2^2 - C_1^2) + g(Z_2 - Z_1)
\]

or as a rate equation

\[
\dot{Q} - \dot{W} = \dot{m}[(h_2 - h_1) + \frac{1}{2}(C_2^2 - C_1^2) + g(Z_2 - Z_1)]
\]
Steady Flow Energy Equation (SFEE)

The SFEE is the general form of 1st law for open systems and is equally applicable to both reversible and irreversible systems.

Implicit assumptions in the SFEE:

- Mass flows at inlet/exit are constant and equal
- The properties at any point do not change with time
- The properties over the cross section at inlet/exit are constant
- Heat and work crossing the boundary do so at a constant rate.
Applications of the SFEE

Many common devices operate with steady flow under normal conditions.

*Constant pressure process:*
Boilers, condensers, heat exchangers, combustion chambers etc.
Applications of the SFEE

The model for these devices is

SFEE:

\[ q - w = (h_2 - h_1) + \frac{1}{2}(C_2^2 - C_1^2) + g(Z_2 - Z_1) \]

hence

\[ q = h_2 - h_1 \quad \text{kJ/kg} \quad \text{or} \quad \dot{Q} = \dot{m}q = \dot{m}(h_2 - h_1) \quad \text{kJ/s (kW)} \]

If the fluid is a perfect gas, i.e. \( pv=RT \), \( du=C_vdT \), \( dh=C_pdT \) etc, then:

\[ q = C_p(T_2 - T_1) \]

\[ \dot{Q} = \dot{m}C_p(T_2 - T_1) \]

\( q \) is expressed in terms of the temperature of the fluid.
Applications of the SFEE

Model for a system containing two fluids (heat exchanger)

SFEE becomes

\[ \dot{Q} - W = m_a (h_2 - h_1) + \dot{m}_b (h_4 - h_3) + \Delta KE + \Delta PE \]

Hence

\[ \dot{m}_a (h_1 - h_2) = \dot{m}_b (h_4 - h_3) \quad \text{for perfect gases} \]

Then

\[ \dot{m}_a C_{p,a} (T_1 - T_2) = \dot{m}_b C_{p,b} (T_4 - T_3) \]
Applications of the SFEE

**Adiabatic processes** (with no work transfer)

Common devices in this category are nozzles and diffusers.

Nozzles and diffusers are ducts of varying area whose purpose is to accelerate or decelerate the flow.
Applications of the SFEE

SFEE becomes:

\[
q - \omega = (h_2 - h_1) + \frac{1}{2} (C_2^2 - C_1^2) + g(Z_2 \frac{\text{small}}{Z_1})
\]

Hence

\[
\frac{1}{2} (C_2^2 - C_1^2) = h_1 - h_2
\]

In the case of a nozzle, \( C_2 \gg C_1 \), hence:

\[
C_2 = \sqrt{2(h_1 - h_2)}
\]

For a perfect gas:

\[
C_2 = \sqrt{2C_p(T_1 - T_2)}
\]
**Applications of the SFEE**

**Adiabatic processes** (with work)

Turbines, compressors, pumps etc all require an energy flow across the boundary in the form of work.

Turbines:

- a high energy inlet flow (high enthalpy)
  - $\Rightarrow$ converted to kinetic energy within the machine
  - $\Rightarrow$ work at the output shaft then the flow exits with a low kinetic energy
Applications of the SFEE

SFEE becomes

\[
\dot{Q} - w = (h_2 - h_1) + \frac{1}{2} \left( C_2^2 - C_1^2 \right) + g \left( Z_2 - Z_1 \right)
\]

\approx 0

\approx 0

\approx 0

Hence:

\[ w = h_1 - h_2 \]

or

\[ \dot{W} = \dot{m}(h_1 - h_2) \]

For a perfect gas

\[ \dot{W} = \dot{m}C_p(T_1 - T_2) \]

Sometimes the exit velocity from a turbine will not be negligible, then:

\[ w = h_1 - h_2 - \frac{1}{2} C_2^2 \]

\[ \dot{W} = \dot{m}(h_1 - h_2 - \frac{1}{2} C_2^2) \]
Extension to fluid mechanics

The Steady Flow Energy equation is used in fluid mechanics problems with a more general background. Combination with Fluid systems in thermal engineering is one application but there are a number of others. The equation:

\[
\frac{Q}{dm} - \frac{W}{dm} = (u_2 + p_2v_2) - (u_1 + p_1v_1) + \frac{1}{2}(C_2^2 - C_1^2) + g(Z_2 - Z_1)
\]

Can be compared with Bernouilli to further explain terms of a per unit weight representation:

\[
\frac{p_1}{\rho g} + \frac{u_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{u_2^2}{2g} + z_2
\]

\[
\frac{q_w - w_w}{g} = \frac{p_2}{\rho g} + \frac{C_2^2}{2g} + z_2 - \left(\frac{p_1}{\rho g} + \frac{C_1^2}{2g} + z_1\right) + \frac{u_2 - u_1}{g}
\]

Pressure head

Velocity head

Potential head
Convert to fluid mechanics notation

The equation (in thermodynamics notation):

\[
\frac{q_w - w_w}{g} = \frac{p_2}{\rho g} + \frac{C_2^2}{2g} + z_2 - \left( \frac{p_1}{\rho g} + \frac{C_1^2}{2g} + z_1 \right) + \frac{u_2 - u_1}{g}
\]

is more usefully written in fluid mechanics as

\[
p_1 + \frac{1}{2} \rho u_1^2 + \rho z_1 g + \rho q_w - \rho w_w = p_2 + \frac{1}{2} \rho u_2^2 + \rho z_2 g + \rho \Delta e
\]

e.g. pressure drop across a pump

"losses"

More correctly change in internal energy since loss cannot happen within a control volume.
Strictly speaking, the derivation for the SFEE given here is for a streamtube containing an ideal fluid i.e. no velocity variation across the pipe. The viscous drag of the walls acting on the fluid is a loss mechanism which produces a drop in pressure. The velocity in a real pipe as we have shown is a function of position and therefore so is the kinetic energy of fluid elements. The correct kinetic energy is found by integration:

\[
\text{Mass flow through element} = \rho \delta Au
\]

\[
\text{kinetic energy flow through element} = \frac{1}{2} \text{mass flow} \times (\text{velocity})^2
\]

\[
= \frac{1}{2} \rho \delta Au \times u^2
\]

\[
= \frac{1}{2} \rho \delta Au^3
\]
Kinetic energy correction

Total kinetic energy flow = \( \frac{1}{2} \int \rho u^3 \delta A \)

total weight flow = \( \int \rho g u \delta A \)

Therefore: actual kinetic energy per unit weight = \( \frac{1}{2g} \int \rho u^3 \delta A \)

If we determine the mean velocity, according to \( \bar{u} = \int \left( \frac{u}{A} \right) dA \)

Then actual kinetic energy per unit weight = \( \alpha \frac{\bar{u}^2}{2g} \)

where \( \alpha \) is a correction factor (1.058 for Prandtl’s law)
Using the energy equation

Diagrammatical representations are useful in a range of applications:

Fan pump:

Turbine generator

$p$

$head$

$x$

$x$
Using the energy equation

This second example is a useful way of examining the importance of the control volume:

\[
p_1 + \frac{1}{2} \rho u_1^2 + \rho z_1 g + \rho q_w - \rho w_w = p_2 + \frac{1}{2} \rho u_2^2 + \rho z_2 g + \rho \Delta e
\]

\[
p_1 + \frac{1}{2} \rho u_1^2 + \rho z_1 g + \Delta p_{\text{input}} - \Delta p_{\text{output}} = p_2 + \frac{1}{2} \rho u_2^2 + \rho z_2 g + \Delta p_{\text{F+S}}
\]

- Pressure rise in a pump (energy in)
- Pressure drop in a turbine (energy out)
- Friction, separation, turbulence losses
Using the energy equation

Further:

\[
p_1 + \frac{1}{2} \rho u^2 + \rho z_1 g + \Delta p_{\text{input}} - \Delta p_{\text{output}} = p_2 + \frac{1}{2} \rho u^2 + \rho z_2 g + \frac{1}{2} \rho K u^2
\]

Cross-section and velocity constant

As we discussed, frictional losses are of this form with \( K \) a constant

NOTE: this neglects local kinetic fluctuations due to surface velocities in the reservoirs (reservoirs are very large).
Using the energy equation

So far the equations are general. For the example shown (with no pump or turbine), the appropriate control volume is that bounded by the free surfaces of the water in the two reservoirs. The energy equation is then

\[ p_1 + \frac{1}{2} \rho u^2 + \rho z_1 g = p_2 + \frac{1}{2} \rho u^2 + \rho z_2 g + \frac{1}{2} \rho Ku^2 \]

If the two reservoirs are open to atmosphere, in the gauge frame of reference:

\[ \rho g (z_1 - z_2) = \frac{1}{2} \rho Ku^2 \]
Energy diagrams

Definition of *hydraulic line* with a graphical representation of head-line along which the water would rise to if an open stand pipe were inserted i.e. Total head minus velocity head.
Definition of *siphon* serves to illustrate the usefulness of this type of diagram. If the height of the pipe rises above the hydraulic line then the pressure in it is below atmospheric pressure, which may lead to gas bubble formation.
The Pitot tube

(a) Pitot tube.

(b) Static tube.

(c) Pitot-static tube.
The Pitot tube

From Bernouilli:

\[
\frac{u^2}{2g} + \frac{p}{\rho g} = \frac{u_0^2}{2g} + \frac{p_0}{\rho g}
\]

but the flow is zero in the static pipe

\[
\frac{p_0}{\rho g} = \frac{u^2}{2g} + \frac{p}{\rho g}
\]

also,

\[
\frac{p}{\rho g} = z \quad \text{and} \quad \frac{p_0}{\rho g} = h + z
\]

So that

\[
\frac{u^2}{2g} = \frac{p_0}{\rho g} - \frac{p}{\rho g} = h
\]

and velocity at A

\[
u = u = \sqrt{2gh}
\]
Venturi meter

- area: $A_1$
- velocity: $u_1$
- pressure: $p_2$

- area: $A_2$
- velocity: $u_2$
- pressure: $p_2$

Converging cone
Diverging throat
Assume no loss of energy, Bernouilli:

\[ z_1 + \frac{p_1}{\rho g} + \frac{u_1^2}{2g} = z_2 + \frac{p_2}{\rho g} + \frac{u_2^2}{2g} \]

\[ u_2^2 - u_1^2 = 2g \left[ \frac{p_1 - p_2}{\rho g} + (z_1 - z_2) \right] \]

For continuous flow: \[ A_1 u_1 = A_2 u_2 \Rightarrow u_2 = \frac{A_1}{A_2} u_1 \]

So that \[ u_1^2 \left[ \left( \frac{A_1}{A_2} \right)^2 - 1 \right] = 2g \left[ \frac{p_1 - p_2}{\rho g} + (z_1 - z_2) \right] \]

or: \[ u_1 = \frac{A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2g \left[ \frac{p_1 - p_2}{\rho g} + (z_1 - z_2) \right]} \]
Venturi meter

Volumetric flow rate:

\[ Q = A_1 u_1 = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2gH} \]

where:

\[ H = \frac{p_1 - p_2}{\rho g} + (z_1 - z_2) \]

often written as

\[ Q = \frac{A_1}{\sqrt{m^2 - 1}} \sqrt{2gH} \]

where \( m = \frac{A_1}{A_2} \)
Venturi meter

The value of $H$ can be found by reading the u-tube:

Pressures across dotted line equal:

$$p_z = p_1 + \rho g (z_1 - z) = p_2 + \rho g (z_2 - z - h) + \rho_{man} gh$$

Rearranging:

$$\frac{p_1 - p_2}{\rho g} + (z_1 - z_2) = h \left( \frac{\rho_{man}}{\rho} - 1 \right) = H$$

So that

$$Q = \frac{A_1}{\sqrt{m^2 - 1}} \sqrt{2gh \left( \frac{\rho_{man}}{\rho} - 1 \right)}$$

which is independent of $z$ and therefore Inclination.
Stream power

A stream of fluid does work according to with total energy per unit weight:

\[ H = \frac{p}{\rho g} + \frac{u^2}{2g} + z \]

If weight per unit time is known then the power is:

\[ \text{energy per unit time} = \text{weight per unit time} \times \text{energy per unit weight} \]

with weight per unit time = \( \rho gQ \) so that:

\[ \text{power} = \rho gQH = \rho gQ \left( \frac{p}{\rho g} + \frac{u^2}{2g} + z \right) = \rho Q + \frac{1}{2} \rho u^2 Q + \rho gQz \]