Q 1. In terms of the cut-off frequency $\omega_0$, obtain the transfer function of a fourth order low pass Butterworth filter.

Q 2. It is required to construct a low-pass Butterworth filter having a cut-off frequency of 1 KHz. Determine the pole locations and hence the transfer-functions for both a third and fourth order filter. Write each transfer-function in both polynomial and quadratic factored forms. Compare the attenuation of the two filters (in dB) at a frequency equal to twice the cut off frequency.

Q 3. A low-pass Butterworth filter is to have a cut-off frequency of 20 Hz. Its attenuation at three times the cut-off frequency is to be at least 25 dB. Obtain the transfer-function of the lowest order filter that will meet this specification.

Q 4. In terms of the cut-off frequency $\omega_0$, obtain the transfer-function of a fourth order high pass Butterworth filter.

Q 5. Obtain the transfer-function of a third order high-pass filter with cut-off frequency of 200 Hz. Evaluate the magnitude of this function at frequencies equal to twice and half the cut-off frequency respectively.

Q 6. A so-called baseband signal has frequency spectrum

$$ X(f) = \text{rect} \left( \frac{f}{400} \right) (1 - \left| \frac{f}{200} \right|) $$

Calculate the Nyquist sampling rate for this signal. Assuming this signal is instantaneously sampled at $f_s = 500$ Hz, sketch the amplitude spectrum of the sampled signal. Repeat this last part for $f_s = 300$ Hz.

Q 7. Obtain the discrete Fourier transform of the following the signal $x_1[n]$ which is zero except for

$$ x_1[-2] = 1, x_1[-1] = 2, x_1[0] = 3, x_1[1] = 2, x_1[2] = 1 $$

Repeat this calculation for the signal $x_2[n]$ which is zero except for

$$ x_2[-3] = -1, x_2[-2] = -1, x_2[-1] = 1, x_2[0] = 1, x_2[1] = -1, x_2(2) = -1, x_2[3] = -1 $$
Q 8. A discrete linear time-invariant system with input $x[n]$ and output $y[n]$ is described by the difference equations where $q[n]$ is an intermediate variable

\[
\begin{align*}
y[n] &= q[n] + \beta q[n - 1] \\
q[n] &= x[n] + \alpha q[n - 1]
\end{align*}
\]

Obtain the difference equation relating $x[n]$ and $y[n]$ and hence show that the impulse response of this system is given by

\[h[n] = \delta[n] + x_s[n - 1](\alpha + \beta)a^{n-1}\]

Q 9.

A discrete linear time-invariant system with input $x[n]$ and output $y[n]$ is described by the following difference equations where $q[n]$ is an intermediate variable

\[
\begin{align*}
y[n] &= 2q[n] + q[n - 1] \\
q[n] &= x[n] + 0.5q[n - 1]
\end{align*}
\]

Using the $z$ transform determine the following for this system:

i) the zero input response given $q[-1] = 2$
ii) the impulse response $h[n]$
iii) the step response.

Q 10. Determine the transfer-function $G(z)$ corresponding to the impulse response

\[g[n] = 100(0.5)^n - (0.5)^n(96 \cos (0.6435n) + 8 \sin (0.6435n))\]