Digital Modulation

• Analogue modulation techniques include amplitude modulation (AM), frequency modulation (FM), and phase modulation (PM);

• equivalent digital modulation forms exist: amplitude shift keying (ASK), frequency shift keying (FSK), or phase shift keying (PSK);

• a large number of other digital modulations are in use, and often combinations are employed;

• we will consider quadrature amplitude modulation (QAM), which can be used for ASK, PSK and A/PSK.
Quadrature Amplitude Modulation

$\cos(\omega t)$

$\sin(\omega t)$

$s(t)$

$\sum \delta(t-kT_s)$

QAM symbol generation

D/A conversion

QAM modulation
Quadrature Amplitude Demodulation

\[ \cos (\omega t) \]
\[ \sin (\omega t) \]
\[ \sum \delta(t-kT_s) \]
Pulse Shaping I

- For analogue transmission, discrete samples have to be pulse shaped:

\[
x(t) = \sum_{k=-\infty}^{+\infty} x(k) \cdot r(t - kT)
\]

\[
x[k] \quad \times \quad r(t) \quad x(t)
\]

\[\Sigma \delta(t-kT)\]

- A number of choices for \( r(t) \) would allow to retrieve the original data sample \( x(k) \) from \( x(t) \): impulse, pulse, sinc function (others?) — which one are best?
• Note: all filters have regular zero-crossing (=Nyquist system), but different support.
• Some filters produce considerable excess bandwidth beyond the symbol rate $f_s$. 
Pulse Shaping IV

- Example: the peak of different shifted sinc functions coincide with zero crossings of all other sincs:
Transmit and Receive Filters

- Pulse shaping has to fulfill a compromise between the filter time duration (symbols are spread over this interval) and bandwidth (limitations imposed on neighbouring channels);

- unlike the extremes of impulse/pulse and sinc function, raised-cosine, Kingsbury filter, and similar (near) Nyquist systems offer this compromise and hence are usually employed;

- the Nyquist system $r(t)$ is separation into a transmit filter $g(t)$ and receive filter $g(-t)$ (square-root Nyquist systems);

- this division of $r(t)$ fulfills two purposes: limitation of the transmitted bandwidth (RX) and suppression of out-of-band noise and possibility to recover the correct sample values (TX);

- the filter $g(-t)$ in the receiver is also called a matched filter.
Digital Amplitude Modulation

• the modulation section in the QAM scheme sketched on slide 46 and 47:

\[
\cos(\omega t) \\
\sin(\omega t)
\]

\[
x_i(t) \\
x_q(t)
\]

\[
s(t) \\
\hat{s}(t)
\]

\[
\hat{x}_i(t) \\
\hat{x}_q(t)
\]

LP

• all signals here are analogue; digital-to-analogue conversion could also take place on the signal \(s(t)\) ("software radio", working digitally up to an intermediate frequency); similarly, the received signal would be directly supplied to an analogue-to-digital converter.
QAM — Modulation

- Modulation of “inphase” and “quadrature” components to carrier frequency $\omega_c$:

\[
\begin{align*}
x_1(t) &= x_i(t) \cdot \cos(\omega_c t) \\
x_2(t) &= x_q(t) \cdot \sin(\omega_c t)
\end{align*}
\]

- the transmitted signal is $s(t) = x_1(t) + x_2(t)$;

- to explain the demodulation, we assume perfect transmission $\hat{s}(t) = s(t)$. 
QAM — Demodulation

• Demodulation example for the “inphase” component:

\[
\hat{x}_i'(t) = s(t) \cdot \cos(\omega_c t) = (x_i(t) \cdot \cos(\omega_c t) + x_q(t) \cdot \sin(\omega_c t)) \cdot \cos(\omega_c t)
\]

\[
= x_i(t) \cdot \cos^2(\omega_c t) + x_q(t) \cdot \sin(\omega_c t) \cos(\omega_c t)
\]

\[
= x_i(t) \cdot \frac{1}{2} \cdot (1 + \cos(2\omega_c t)) + x_q(t) \cdot \frac{1}{2} \cdot \sin(2\omega_c t)
\]

• if the lowpass filter LP in slide 46 is selected appropriately, the components modulated up to frequency \(2\omega\) can be filtered out (cut-off frequency \( \leq \omega_c \)); hence:

\[
\hat{x}_i(t) = \text{LP}(\hat{x}_i'(t)) = \frac{1}{2} x_i(t)
\]

• a similar calculation can be performed for the demodulation of \(\hat{x}_q(t)\):

\[
\hat{x}_q'(t) = \cdots = x_i(t) \cdot \frac{1}{2} \cdot \sin(2\omega_c t) + x_q(t) \cdot \frac{1}{2} \cdot (1 - \cos(2\omega_c t))
\]
Modulation — Complex Notation I

- The previous modulation / demodulation scheme can also be expressed in complex notation: “inphase” and “quadrature” components are real and imaginary part of the signal:

\[ x(t) = x_i(t) + j \cdot x_q(t) \]

- The transmitted signal is obtained by taking the real part only:

\[ s(t) = \text{Re}\{x(t) \cdot e^{-j\omega_c t}\} \]

- Flow graph:
Modulation — Complex Notation II

• Modulation:

\[ v(t) = e^{-j\omega_c t} \cdot x(t) \]
\[ = \left( \cos(\omega_c t) - j \sin(\omega_c t) \right) \cdot \left( x_i(t) + j \cdot x_q(t) \right) \]
\[ = x_i(t) \cdot \cos(\omega_c t) + x_q(t) \cdot \sin(\omega_c t) - j x_i(t) \cdot \sin(\omega_c t) + j x_q(t) \cdot \cos(\omega_c t) \]

\[ \text{real} \quad \text{imaginary} \]

• transmitted signal:

\[ s(t) = \Re \{ v(t) \} = x_i(t) \cdot \cos(\omega_c t) + x_q(t) \cdot \sin(\omega_c t) \]

• this is identical to the signal \( s(t) \) on slide 10.
Demodulation — Complex Notation

- flow graph for the complex demodulation scheme:

\[ \hat{s}(t) \rightarrow \hat{x}'(t) \rightarrow \text{LP} \rightarrow \hat{x}(t) \]

\[ e^{j\omega t} \]

- the demodulated signal:

\[ \hat{x}'(t) = e^{j\omega t} \cdot s(t) \]

\[ = (\cos(\omega_c t) + j \sin(\omega_c t)) \cdot (x_i(t) \cdot \cos(\omega_c t) + x_q(t) \cdot \sin(\omega_c t)) \]

\[ = x_i(t) \cdot \frac{1}{2} (1 + \cos(2\omega_c t) + j \sin(2\omega_c t)) + \]

\[ + j x_q(t) \cdot \frac{1}{2} (1 - \cos(2\omega_c t) - j \sin(2\omega_c t)) \]

- the lowpass filter (LP) will again remove components modulated at 2\(\omega\).
Bits to Symbols

• the bit stream to be transmitted is serial / parallel multiplexed onto a stream $z(k)$ of symbols of $q$ bits (discrete $2^q$ levels);

• example for $q = 2$ bit symbols:
Mapping to Constellation Pattern

- the symbols \( z(k) \) are translated into values for the inphase and quadrature components, \( x_i(k) \) and \( x_q(k) \), by assigning amplitude values in a constellation pattern;

- example for a case of \( q = 2 \) bit symbols:

- from the constellation pattern, the amplitudes \( x_i(k) \) and \( x_q(k) \) are determined;

- in the receiver, the constellation point (and therefore the transmitted symbol) is determined from \( \hat{x}_i(k) \) and \( \hat{x}_q(k) \).
Phase Shift Keying (PSK)

- Phase shift keying (PSK): example for constellation pattern and transmitted signal $s(t)$ (with rectangular pulse shaping):

- here, each symbol period holds three cycles of the carrier; pulse shaping would smooth discontinuities in the signal.
Amplitude Shift Keying (ASK)

- Amplitude shift keying (ASK): example for constellation pattern and transmitted signal $s(t)$ (with rectangular pulse shaping):

\[ i(k) \times q(k) \times x(k) \times k(0,0)(0,1)(1,0)(1,1) \rightarrow \text{time } t \]

- note: (i) the quadrature component is not used; (ii) this is not purely ASK, as a phase shift of $\pi$ is exploited in the modulation scheme.
Combined ASK / PSK

- QAM multilevel modulation schemes are possible, combining features of PSK and ASK. An example of 16-QAM:

- depending on the channel quality, 64-QAM, 128-QAM, or 256-QAM are possible.
Gray Mapping

- If noise or distortions are likely to cause misclassification in the receiver, Gray code mapping can minimize the bit error rate:

  - adjacent constellation points only vary in one single bit (minimum Hamming distance).
**Eye Diagram — Perfect Channel**

- We are looking at stacked 2 symbol period intervals of the demodulated signal $\hat{x}_i(t)$ in a QPSK scheme:

![Eye Diagram](image)

- this is also called an eye diagram; ideal sampling of $\hat{x}_i(k)$ will sample the crossing points $\hat{x}_i(t) = \pm 1 \rightarrow$ clock/timing recovery.
Eye Diagram — Noisy Channel

• With channel noise at dB SNR, the eye diagram looks different:

• important: as long as the sampling points can be clearly determined and the eye is “open”, $\hat{x}_i(k)$ will correctly resemble $x_i(k)$. At higher noise levels, misclassifications can occur if the eye is “closed”.
The channel is now non-ideal with an impulse response \( c(t) = \delta(t) - \frac{1}{2} \cdot \delta(t - T/4) \), where \( T \) is the symbol period:

- the eye diagram is distorted; more severely distorting channels will lead to misclassification due to intersymbol interference.
Intersymbol Interference (ISI)

- Consider the response of an ideal pulse shaping filter with regular zero crossings, and the same system in combination with the channel impulse response $c(t)$:

- the system Tx-filter – channel – Rx filter has lost the property of a Nyquist system; peaks of the function no longer coincide with zero crossings of neighbouring pulses.
Synchronisation

- The process of selecting the correct sampling instances is called synchronisation (or timing recovery);

$$kT_s - \tau$$

$$\hat{x}(t) \rightarrow \hat{x}[k]$$

- this is equivalent to replacing the impulse train $$\sum \delta(t - kT_s)$$ in slide 2 by $$\sum \delta(t - kT_s - \tau)$$ with $$0 \leq \tau \leq T_s$$;

- the eye diagrams on slides 21 – 23 gave some indication on where to sample the signal (example: on slide 21, $$\tau \approx 0.85 \cdot T_s$$);

- usually, the received symbols are oversampled, and from the distribution (histogram) of the sample sets for different $$\tau$$, the one with the smallest deviation from discrete levels (depending on the QAM mode — 16-QAM, 64-QAM, etc.) is chosen;

- this also permits to track clock mismatches between Tx and Rx!
Carrier Offset

• Consider demodulation whereby the carrier is not exactly adjusted but has a frequency offset $\Delta \omega$ and a phase offset $\varphi$.

\[
\hat{s}(t) \rightarrow \hat{x}'(t) \rightarrow \text{LP} \rightarrow \hat{x}(t)
\]

\[
e^{j((\omega_c + \Delta \omega)t + \varphi)}
\]

• if no channel distortion is present, i.e. $\hat{s}(t) = s(t)$, then the demodulated QAM signal prior to sampling is $\hat{x}(t) = x(t) \cdot e^{j(\Delta \omega t + \varphi)}$.

• effect: rotation of the demodulated constellation pattern; this needs to be compensated in the receiver.
Equalisation

- If the channel has severe amplitude and phase distortion, sampling alone is unable to recover the correct symbols — equalisation is required:

\[
\begin{align*}
X(z) & \rightarrow C(z) \rightarrow W(z) \rightarrow \hat{X}(z)
\end{align*}
\]

- the system \( C(z) \) is a baseband model of the channel (including modulation, physical transmission channel, and demodulation);

- we want to find an equalisation filter \( W(z) \) such that the recovered symbols \( \hat{X}(z) \) are only delayed versions of the transmitted signal, \( \hat{X}(z) = z^{-\Delta} \cdot X(z) \);

- the (in the least squares sense) optimal solution for the noise-free case is

\[
W(z) \cdot C(z) = z^{-\Delta} \quad \text{or} \quad W(z) = z^{-\Delta} \cdot C^{-1}(z)
\]
Conclusions

• We have defined all major blocks in the digital communications chain:

  input
  source encoding → channel encoding → modulation → channel →
  output
  source decoding → channel decoding → demodulation

• Your assignment will be mostly based on the implementation of the modulation part, with modulation, optimal filtering, and the possibility to recover the transmitted symbols;

• Although not required for the assignment, it is worthwhile to be aware of the general ideas of source and channel coding.