Q 1. A so-called baseband signal has frequency spectrum

\[ X(f) = \text{rect}(\frac{f}{400})(1 - \frac{|f|}{200}) \]

Calculate the Nyquist sampling rate for this signal. Assuming this signal is instantaneously sampled at \( f_s = 500 \text{ Hz} \), sketch the amplitude spectrum of the sampled signal. Repeat this last part for \( f_s = 300 \text{ Hz} \).

Q 2. Obtain the discrete Fourier transform of the following the signal \( x_1[n] \) which is zero except for

\[ x_1[-2] = 1, x_1[-1] = 2, x_1[0] = 3, x_1[1] = 2, x_1[2] = 1 \]

Repeat this calculation for the signal \( x_2[n] \) which is zero except for

\[ x_2[-3] = -1, x_2[-2] = -1, x_2[-1] = 1, x_2[0] = 1, x_2[1] = -1, x_2[3] = -1 \]

Q 3. A discrete linear time-invariant system with input \( x[n] \) and output \( y[n] \) is described by the difference equations where \( q[n] \) is an intermediate variable

\[
\begin{align*}
y[n] &= q[n] + \beta q[n-1] \\
q[n] &= x[n] + \alpha q[n-1]
\end{align*}
\]

Obtain the difference equation relating \( x[n] \) and \( y[n] \) and hence show that the impulse response of this system is given by

\[ h[n] = \delta[n] + x_s[n-1](\alpha + \beta)\alpha^{n-1} \]

Q 4.

A discrete linear time-invariant system with input \( x[n] \) and output \( y[n] \) is described by the following difference equations where \( q[n] \) is an intermediate variable

\[
\begin{align*}
y[n] &= 2q[n] + q[n-1] \\
q[n] &= x[n] + 0.5q[n-1]
\end{align*}
\]

Using the z transform determine the following for this system:

i) the zero input response given \( q[-1] = 2 \)
ii) the impulse response \( h[n] \)
iii) the step response.
Q 5. Determine the transfer-function \( G(z) \) corresponding to the impulse response
\[
g[n] = 100(0.5)^n - (0.5)^n(96\cos(0.6435n) + 8\sin(0.6435n))
\]

Q 6. A discrete linear time-invariant system has frequency response
\[
G(e^{j\omega}) = e^{-j(\omega + \frac{\pi}{4})} \left( \frac{1 + e^{-j2\omega} + 4e^{-j\omega}}{1 - \frac{1}{2}e^{-2j\omega}} \right), -\pi \leq \omega \leq \pi
\]
Determine the output \( y[n] \) for all \( n \) in response to the input
\[
x[n] = \cos\left(\frac{3\pi n}{2}\right)
\]

Q 7. Which of the following discrete linear systems are causal? Justify your answer in each case
i) \( y[n] = c_1 n[n + 1] + c_0 x[n] \).
ii) \( y[n] = a(n)x[n] \).
iii) \( y[n] = a(n + 1)x[n] \).
iv) \( y[n] = x[2n] \).

Q 8. In calculating the frequency response, \( X(\omega) \), of a linear time-invariant bandpass filter whose impulse is aperiodic (i.e. non periodic), a designer obtains an \( X(\omega) \) that is periodic with period 2\( \pi \) and
\[
X(\omega) = 0, \text{ for } |\omega - n\pi| < 0.1\pi, \ n = 0, \pm 1, \pm 2, \ldots
\]
Which, if any, of the following statements are correct? Give your answer as ‘true’ or ‘false’ and give reasons.
(i) The features of the result are to be expected.
(ii) The designer should check his/her calculation since the Fourier transform of an aperiodic discrete time signal is not periodic.
(iii) The designer has made an error because, for a band-pass filter, \( X(\omega) \) should equal to zero for large values of \( |\omega| \).
(iv) If the impulse response were periodic instead of aperiodic, \( X(\omega) \) would be composed of impulses.

Q 9. An FIR filter is given by
\[
G(z) = \frac{1}{4} + \frac{1}{2} z^{-1} + \frac{1}{4} z^{-2}
\]
where the sampling time is 10\(^4\) seconds. Calculate the amplitude and phase responses of this filter as a function of frequency and sketch both of them. Calculate the group
delay of this filter. Determine the frequency at which the response of the filter is \(-3\, \text{dB}\).

Suppose also that the filter is cascaded in series with itself to form

\[
\hat{G}(z) = G(z)G(z)
\]

What is the frequency of the pass-band edge of this new filter?

Q 10. A difference equation relating the input \((x)\) and output \((y)\) of a discrete-time linear time-invariant system is given by

\[
y[n] = x[n] - a^{25}x[n - 25] - ay[n - m]
\]

What is the essential difference between i) selecting \(m\) as a positive integer and ii) selecting it as a negative integer? Sketch an architecture to implement this difference equation for the cases when \(m = 1\) and \(m > 1\) respectively. This should use adders, delay elements and the minimum number of multipliers.

For the difference equation of above, show that

\[
y[n] = \sum_{k=0}^{24} a^k x[n - k]
\]

and identify the form of filter this difference equation represents.

Find the \(z\)-transfer-function of the difference equation above. Find also the d.c. gain of this filter.

Q 11. A signal has been sampled at 0.0025 sec and examination reveals a disturbance with a frequency of 50 Hz. Design an FIR notch filter with two poles and two zeros suitably placed in the complex plane to filter out this disturbance. The filter is required to produce real signals for a real input. Determine the transfer-function of the filter and sketch its frequency response.