Current & Resistance

- Current & current density
- Ohm’s law (microscopic)
- Ohm’s law (macroscopic)

https://cosmolearning.org/video-lectures/ohms-law-resistivity-currents/
Current and current density

- Current: \( I[A] = \frac{Q[C]}{t[s]} \)
  - positive particle flow

\[
i = \frac{dq}{dt} \quad q = \int_0^t dq = \int_0^t i \, dt
\]

- Current density \( J = \frac{I[A]}{A[m^2]} \)

\[
i = \int \vec{J} \cdot d\vec{A} = JA
\]
Drift Velocity

- Relation between I and V
- Relation between J and E
  - J=I/A and E=V/d

- Infinite acceleration?
  - Yes: superconductor
  - No: scattering

- Average time between collision $\tau$
  - Statistical average!
  - Independent of Drift velocity!
  - Thermal/Diffusion velocity much larger

\[
F = ma = qE
\]
\[
a = eE / m
\]
\[
v_d = a = \frac{eE}{m}
\]
\[
v_{th} = 10^6 \text{ m} / \text{s} \quad v_d = 10^{-4} \text{ m} / \text{s}
\]
Current, Velocity

- Moving particle in a medium
- Relation between velocity and current

\[ n = \text{number of charges } e \text{ per unit volume} \]

\[ Q = neAd = \text{total mobile charge in length } d \text{ of the conductor} \]

\[ t = \frac{d}{V_d} = \text{time for this charge to sweep past the current measuring point} \]

\[ I = \frac{Q}{t} = \frac{neAd}{d/V_d} \]

\[ I = neA V_d \]
Ohm’s law (microscopic)

\[ v_d = a \tau = \frac{eE \tau}{m} \]

\[ i = enA_d \]

\[ J = \frac{i}{A} \]

\[ J = en v_d = \frac{ne^2 E}{m} \tau \]

- Conductivity \( \sigma \)

\[ J = \sigma E \quad \sigma = \frac{ne^2 \tau}{m} \]
Ohm’s law (macroscopic)

\[ J = \sigma E \quad \sigma = \frac{ne^2 \tau}{m} \]

- Relation between \( I \) and \( V \)
- Resistivity \( \rho \) \( \rho = 1/\sigma \)

\[ i = JA \quad V = EL \quad i = \sigma \frac{AV}{L} \quad i = \frac{AV}{\rho L} \]

- Resistance \( R \)

\[ R = \rho \frac{L}{A} = \frac{V}{i} \]
Ohm’s law (macroscopic)

- What is Ohm’s law?
- $I \propto V$
  - Polarity and direction
  - Not always true!
    - Superconductors (no resistance)
    - Asymmetric materials ($\sigma$ is a tensor)
- Ohmic material: satisfies Ohm’s law
- Ohmic contact: voltage drop negligible
  - Not diodes!
Power (Lines)

- Power $P$ (rate of energy transfer)
  \[ U = Vdq = VIdt \quad P_{tr} = \frac{dU}{dt} = VI \]

- Resistive dissipation (power loss)
  \[ P_{loss} = I^2R \quad \frac{P_{loss}}{P_{tr}} = \frac{IR}{V} \]

- High voltage gives lower losses
  - $I=\Delta V/R$ not $I=V/R$!
  - In resistive circuit $\Delta V=V$; once at ground no power left
  - In power lines $\Delta V<<V$; losses are very small

- How to get high voltage?
  - Transformer: AC voltage required
Definitions

- Scattering time $\tau$
- Conductivity $\sigma$ 
  \[ \sigma = \frac{ne^2\tau}{m} = ne\mu \]
- Resistivity $\rho$ 
  \[ \rho = \frac{1}{\sigma} \]
- Mobility $\mu$ 
  \[ = \frac{e}{m} \]
Resistance (Temperature)

- Temperature coefficient of metal resistance
  - Scattering time decreases
  - Resistance increases

\[ R = R_0 \left( 1 + \alpha (T - T_0) \right) \]

\[ TCR = \alpha [10^{-3} K^{-1}, \text{ ppm} / K] \]

<table>
<thead>
<tr>
<th>metal</th>
<th>( \rho \ [10^{-9} \Omega m] )</th>
<th>( \alpha \ [10^{-3}/^\circ C] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cu</td>
<td>17</td>
<td>4.0</td>
</tr>
<tr>
<td>Al</td>
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</tr>
<tr>
<td>CuNi(Mn)</td>
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<td>0.02</td>
</tr>
<tr>
<td>Si (pure)</td>
<td>&gt;1G</td>
<td>(-75)</td>
</tr>
</tbody>
</table>

- Temperature dependence of semiconductor resistance
  - Carrier concentration increases
  - Resistance decreases (not linear)
Figure 1 shows an electric cable with a core Cu conductor, a dielectric material, and a conducting sheath around it. It is rated at 600MW power transmission at a potential of 400kV.

(a) Use Gauss' law to derive an expression for the electric field within the dielectric material. [5 marks]

(b) Draw a graph of the potential and electric field against radial distance. [5 marks]

(c) Determine the capacitance per unit length of the system, given that \(a=20\text{mm}\) and \(b=60\text{mm}\) and \(\varepsilon_r=3\). [5 marks]

(d) Calculate the maximum electric field in the dielectric. [5 marks]

(e) Calculate the power loss in the cable given the resistivity of the Cu core is 17 n\(\Omega\)m and the length of the cable is 500km. [5 marks]