ELEC 1206 Electrical Materials and Fields

Electric Potential

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Electric Potential

- Potential and potential energy
  - Equipotential surfaces
  - Calculating potentials
  - Spherical Conductors
- Potential gradient: electric field

https://cosmolearning.org/video-lectures/electric-energy/
Electric Potential Energy

- see Ch 7 and 8 of “Halliday Resnick Walker”
  - kinetic energy and potential energy
  - work \( W = -\Delta U \): change in potential energy
  - if reversible: conservative force
  - work is path independent

\[
\Delta U = - \int_i^f F(x) \, dx
\]

- electrostatic:

\[
\Delta U = - \int_i^f qE(x) \, dx - \int_i^f E(x) \, dx = dV
\]

\[
\Delta U = qdV
\]
## Gravitation and Electricity

<table>
<thead>
<tr>
<th>Gravitation</th>
<th>Electrostatic</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>law of Physics:</strong></td>
<td><strong>Field</strong></td>
</tr>
<tr>
<td>$\vec{F} = G \frac{m_1 m_2}{r^2}$</td>
<td>$\vec{F} = k \frac{q_1 q_2}{r^2}$</td>
</tr>
<tr>
<td><strong>Field</strong></td>
<td></td>
</tr>
<tr>
<td>$g = \frac{F}{m}$</td>
<td>$E = \frac{F}{q}$</td>
</tr>
<tr>
<td><strong>Potential Energy</strong></td>
<td></td>
</tr>
<tr>
<td>$- \int_i^f m g(z) \delta z$</td>
<td>$- \int_i^f q E(x) \delta x$</td>
</tr>
<tr>
<td><strong>in uniform field</strong></td>
<td></td>
</tr>
<tr>
<td>$mgh$</td>
<td>$qEd = qV$</td>
</tr>
<tr>
<td><strong>Equipotential lines</strong></td>
<td><strong>height</strong></td>
</tr>
<tr>
<td></td>
<td><strong>voltage</strong></td>
</tr>
<tr>
<td><strong>Force/Field Lines</strong></td>
<td><strong>slope</strong></td>
</tr>
<tr>
<td>$- \frac{\partial z}{\partial y}$</td>
<td>$E(y) = - \frac{\partial V}{\partial y}$</td>
</tr>
</tbody>
</table>
Equipotential surfaces

- Equipotential surface
- Field line
Equipotential surfaces

http://www.cco.caltech.edu/~phys1/java/phys1/EField/EField.html
Equipotential surfaces

http://www.cco.caltech.edu/~phys1/java/phys1/EField/EField.html
Electric Potential

- Work is path independent

\[ - \int_{i}^{f} E(x) \delta x = V_f - V_i \]

\[ - \int_{b}^{b} E(x) \delta x = V_b - V_b = 0 \]

\[ \oint E(x) \delta x = 0 \]

\[ \oint \vec{E}. \delta \vec{s} = 0 \]
Single Charge

- Potential at point \( P \) ?
- Potential at infinity = 0
  - Potential is relative
- Move charge from infinity:

\[
V_f - V_i = -\int_{\infty}^{R} E dr
\]

\[
\vec{E} = \frac{Q}{4\pi\varepsilon_0 \vec{r}^2}
\]

\[
V_f = -\frac{Q}{4\pi\varepsilon_0} \int_{\infty}^{R} \frac{1}{r^2} dr = \frac{Q}{4\pi\varepsilon_0} \left[ \frac{1}{r} \right]_{\infty}^{R} = \frac{Q}{4\pi\varepsilon_0 R}
\]
Point Charge

\[ V_f = -\frac{Q}{4\pi\varepsilon_0} \int_0^R \frac{1}{r^2} dr = \frac{Q}{4\pi\varepsilon_0} \left[ \frac{1}{r} \right]_0^R = \frac{Q}{4\pi\varepsilon_0 R} \]

- point charge: no radius: \( R=0 \)
  - Infinite potential:
  - Physically impossible
  - but ok to deal with mathematically
    - if not too close
Spherical Conductor

- All charges on outside shell:
  - Electric Field inside conductor?
    - \( E = 0 \) (we cannot draw field lines)
    - \( V = \) constant (integral of electrical field)
  - Electric Field outside conductor?
    - Newton shell’s theorem (Ch. 13.2)
      - as if all charge is in the centre of sphere
      - same as point charge
- All charges uniformly distributed?
  - Impossible in a conductor!

\[
V_f - V_i = -\int_{\infty}^{R} Edr \quad \vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}
\]
Isolated Conductor

- No Current $\Rightarrow$ No Electric Field
- No Electric Field $\Rightarrow$ constant Potential
- Even in an external field!
Electric Field from Potential

- **1D**
  \[-\int_1^f E(x) \, dx = V \quad E(x) = -\frac{\partial V}{\partial x}\]

- **3D**
  \[\vec{E} = -\frac{\partial V}{\partial x} \hat{x} - \frac{\partial V}{\partial y} \hat{y} - \frac{\partial V}{\partial z} \hat{z}\]

- gradient:
  \[\vec{E} = -\nabla V\]
Electric Field from Potential

• Point charge

\[ V_f = \frac{Q}{4\pi\varepsilon_0 r} \]
\[ \vec{E} = \frac{Q}{4\pi\varepsilon_0 \vec{r}^2} \]

\[ E(x) = -\frac{\partial V}{\partial x} \]

\[ E_x = \frac{xQ}{4\pi\varepsilon_0 (x^2 + y^2 + z^2)^{3/2}} \]

\[ E_x(d,0,0) = \frac{-dQ}{4\pi\varepsilon_0 (d^2)^{3/2}} = \frac{-Q}{4\pi\varepsilon_0 d^2} \]

\[ E_x(0,d,0) = 0 \]
Tutorial

• For next lecture

• Problems
  – 24.1  charge and energy
  – 24.2  unit of potential energy
  – 22.4  parallel plates
  – 22.11 non conducting sphere (difficult!)
  – 22.44 symmetry
  – 22.102 energy conservation
1. A particular 12 V car battery can send a total charge of 84 A · h (ampere-hours) through a circuit, from one terminal to the other. (a) How many coulombs of charge does this represent? (Hint: See Eq. 21-3.) (b) If this entire charge undergoes a change in electric potential of 12 V, how much energy is involved?

2. The electric potential difference between the ground and a cloud in a particular thunderstorm is $1.2 \times 10^9$ V. In the unit electron-volts, what is the magnitude of the change in the electric potential energy of an electron that moves between the ground and the cloud?

3. Much of the material making up Saturn’s rings is in the form of tiny dust grains having radii on the order of $10^{-6}$ m. These grains are located in a region containing a dilute ionized gas, and they pick up excess electrons. As an approximation, suppose each grain is spherical, with radius $R = 1.0 \times 10^{-6}$ m. How many electrons would one grain have to pick up to have a potential of $-400$ V on its surface (taking $V = 0$ at infinity)?

sec. 24-5  Calculating the Potential from the Field

4. Two large, parallel, conducting plates are 12 cm apart and have charges of equal magnitude and opposite sign on their facing surfaces. An electrostatic force of $3.3 \times 10^{-15}$ N acts on an electron placed anywhere between the two plates. (Neglect fringing.) (a) Find the electric field at the position of the electron. (b) What is the potential difference between the plates?
11. A nonconducting sphere has radius $R = 2.31$ cm and uniformly distributed charge $q = +3.50$ fC. Take the electric potential at the sphere's center to be $V_0 = 0$. What is $V$ at radial distance (a) $r = 1.45$ cm and (b) $r = R$. (*Hint:* See Section 23-9.)

44. In Fig. 24-48, seven charged particles are fixed in place to form a square with an edge length of 5.0 cm. How much work must we do to bring a particle of charge $+6e$ initially at rest from an infinite distance to the center of the square?

102. (a) A proton of kinetic energy 3.50 MeV travels head-on toward a lead nucleus. Assuming that the proton does not penetrate the nucleus and that the only force between proton and nucleus is the Coulomb force, calculate the smallest center-to-center separation $d_p$ between proton and nucleus when the proton momentarily stops. If the proton were replaced with an alpha particle (which contains two protons) of the same initial kinetic energy, the alpha particle would stop at center-to-center separation $d_\alpha$. (b) What is $d_\alpha/d_p$?