ELEC 1206 Electrical Materials and Fields
Magnetic Field & Lorentz Force

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Magnetic Field

• Electric field was defined as $\vec{E} = \frac{\vec{F}_e}{q}$

• There are no magnetic charges to define the magnetic field in the same way

• 1819 Ørsted – discovered that a magnetic need needle is affected when brought close to a current carrying conductor
Magnetic Field

How is magnetic field defined?

\[ \vec{F}_B \perp \vec{v} \]

\[ F_B \propto v \]

\[ F_B \propto q \]

\[ \vec{F} = q\vec{v} \times \vec{B} \]

Lorentz force

\[ [B] = T \text{ (tesla)} = \frac{Ns}{Cm} \]

https://cosmolearning.org/video-lectures/magnetic-field-lorentz-force/
Total force on a moving charge

\[ \overline{F} = q(\overline{E} + \overline{v} \times \overline{B}) \]

Electric field can do work on a moving charge – can slow or accelerate the charge (change its kinetic energy).

Magnetic field cannot do work on a moving charge (cannot change its kinetic energy) – however it can change the direction of movement, the trajectory of the moving charge.
Lorentz force

\[ \vec{F} = q \vec{v} \times \vec{B} \quad F = q v B \sin \theta \]

\[ \vec{A} \times \vec{B} = (a_y b_z - a_z b_y) \hat{z} + (a_z b_x - a_x b_z) \hat{x} + (a_x b_y - a_y b_x) \hat{y} \]

- B=magnetic flux density:
  - also “magnetic field”
  - defined by Lorentz force
- Velocity/current dependent
  - no current, no magnetic force
  - proportional to velocity
- Cross product (vector):
  - v and B parallel: no force
  - v and B perpendicular: \( F = q v B \)
  - Direction: right hand rule
  - non-commutating
    - \( A \times B \neq B \times A \)
Force on a wire

- Same Lorentz force

\[
\vec{F}_b = q\vec{v} \times \vec{B}
\]

\[
q = neAl \\
\vec{v} = \frac{\vec{I}}{neA}
\]

\[
\vec{F}_b = l\vec{I} \times \vec{B} \\
\vec{F}_b = lIB \sin \phi
\]
Force on a current loop

- Horizontal loop $\alpha=0$
- short end ($\varphi=0$) $F=0$
- long end ($\varphi=90$) $F=\text{upwards}$
  - equal and opposite on each side...
  - but different line of action
  - torque
  - motion
- Loop under any angle $\alpha$
- short end ($\varphi=\alpha$) $F=\text{outwards}$
  - Forces of two sides cancel
- long end ($\varphi=90$) $F=\text{upwards}$
  - Rotation stops when vertical

\[ T = F \frac{W}{2} \cos \alpha = 2 * l \vec{I} \times \vec{B} * \frac{W}{2} = IBA \cos \alpha \]
DC and AC motor

- use commutator or AC current
- switch current direction every half a cycle
- momentum will push in through the top
- force now reversed and loop will rotate continuously

When electric current passes through a coil in a magnetic field, the magnetic force produces a torque which turns the DC motor.
Moving particle in vacuum

\[ \vec{F} = q\vec{E} \quad \vec{F} = q\vec{v} \times \vec{B} \]

- Thomson 1897
- Discovery of electron

\[ \vec{a} = \frac{q(\vec{v} \times \vec{B} + \vec{E})}{m} \]
Moving charged particle in a magnetic field

\[ q_{p,e} = \pm 1.6 \cdot 10^{-19} \text{C} \]
\[ m_p = 1.7 \cdot 10^{-27} \text{kg} \]
\[ m_e = 9.1 \cdot 10^{-31} \text{kg} \]
\[ c = 3 \cdot 10^8 \text{m/s} \]

\[ F = qvB = \frac{mv^2}{R} \]
\[ R = \frac{mv}{qB} \]
\[ R = \sqrt{\frac{2mV}{qB^2}} \]

Numerical example

For p, 1MeV

\[ kE = q\Delta V = \frac{1}{2}mv^2 \]

\[ 1.6 \cdot 10^{-19} \cdot 10^6 = 1.6 \cdot 10^{-13} \text{J} \]
\[ v = \sqrt{\frac{2 \cdot 1.6 \cdot 10^{-13}}{1.7 \cdot 10^{-27}}} = 1.37 \cdot 10^7 \text{ m/s} \]
\[ B = 1 \text{T} \]
\[ R = \frac{1.7 \cdot 10^{-27} \cdot 1.37 \cdot 10^7}{1.6 \cdot 10^{-19} \cdot 1} = 0.1456 \approx 0.15 \text{m} \]
Mass spectrometer

- Uniform circular motion
- Centripetal force/acceleration

\[
\begin{align*}
v &= \frac{T}{4} \int_0 a \cos \theta \, dt = \frac{T}{4} \int_0 a \cos \frac{2\pi t}{T} \, dt = \frac{aT}{2\pi} = \frac{ar}{v} \\
\end{align*}
\]

\[
\begin{align*}
\vec{a} &= \frac{v^2}{r} \\
\vec{a} &= \frac{F}{m} = \frac{q\vec{v}x\vec{B}}{m} \\
\end{align*}
\]

\[
\begin{align*}
m &= \frac{qvBr}{v^2} = \frac{qBr}{v} \\
m &= \frac{qB^2r^2}{2V} \\
\end{align*}
\]

- Photographic plate
- Faraday cups
Hall-effect (1879)

- Positive particle flow
- Apply voltage $V_y$ to generate current through material
- Apply Magnetic Field $B_z$ perpendicular to flow
- Lorentz force moves particles in $y$-direction
  - Builds up an electric field in $y$-direction
  - Until magnetic field and electric field are of equal force

\[
\vec{F}_b + \vec{F}_e = 0
\]

\[
\vec{F}_b = q\vec{v} \times \vec{B} \quad F_{bx} = ev_y B_z
\]

\[
\vec{F}_e = q\vec{E} \quad F_{ex} = eE_x
\]

\[
E_x = -v_y B_z
\]
Hall-effect (cont.)

- Electrons

\[ \vec{F}_b = q \vec{v} \times \vec{B} \quad F_{bx} = (-e)(-v_y)B_z = ev_yB_z \]

\[ \vec{F}_e = q \vec{E} \quad F_{ex} = -eE_x \]

\[ E_x = v_y B_z \]

- Hall effect determines sign of carrier!

\[ V = EW = \pm vBw \]
Hall-effect (final)

\[ V = E_w = \pm v B_w \]

\[ V = \pm v B_w = \pm \frac{I}{neA} B_w = \pm \frac{IB}{ned} \quad A = wd \]

- \( I,B,e,d \) known:
- Hall effect determines
  - Sign of carrier!
  - Number of carriers!
- Ohm’s law + Hall effect
  - Lifetime or mobility

\[ \sigma = \frac{ne^2 \tau}{m} \quad v_d = a \tau = \frac{eE \tau}{m} = \mu E \quad \mu = \frac{e \tau}{m} \]