ELEC 1206 Electrical Materials and Fields

Waves

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Wave Equation

\[ \frac{\partial^2 y(x, t)}{\partial t^2} = \frac{T}{\mu} \frac{\partial^2 y(x, t)}{\partial x^2} \]

Wave equation for a disturbance propagating along a string, which has a tension \( T \) and mass per unit length \( \mu \).

\[ \frac{\partial^2 \nu(x, t)}{\partial t^2} = \frac{1}{LC} \frac{\partial^2 \nu(x, t)}{\partial x^2} \]

Wave equation for a voltage disturbance propagating along an electrical transmission line, \( L \) and \( C \) are inductance and capacitance per unit length.

\[ \frac{\partial^2 E(x, t)}{\partial t^2} = \frac{1}{\varepsilon \mu} \frac{\partial^2 E(x, t)}{\partial x^2} \]

One dimensional \((x)\) wave equation for an electric field disturbance propagating along \( x \) direction in a medium with an electrical permittivity \( \varepsilon \) and a magnetic permeability \( \mu \).
Principle of Superposition for Waves

The Wave Equation

\[
\frac{d^2 y}{dx^2} = \frac{1}{v^2} \frac{d^2 y}{dt^2}
\]

Given two waves

\[ y_1(x, t), y_2(x, t) \]

What would be the resultant wave (net wave)?

\[ y'(x, t) = y_1(x, t) + y_2(x, t) \]

The Principle of Superposition!
Wave Interference

Two waves: same amplitude, same wavelength, same direction

\[ y_1(x, t) = A \sin(kx - \omega t) \]
\[ y_2(x, t) = A \sin(kx - \omega t + \phi) \]

**The resultant wave:**

\[ y'(x, t) = y_1(x, t) + y_2(x, t) \]
\[ y'(x, t) = [2A \cos \frac{\phi}{2}] \sin(kx - \omega t + \frac{1}{2} \phi) \]
Wave Interference

The resultant wave:

\[ y'(x, t) = [2A\cos\frac{1}{2}\phi]\sin(kx - \omega t + \frac{1}{2}\phi) \]
Wave Interference

Two waves: same wavelength, same direction, different amplitude,

\[ y_1(x, t) = A_1 \sin(kx - \omega t) \]
\[ y_2(x, t) = A_2 \sin(kx - \omega t + \phi) \]

The resultant wave:

\[ y'(x, t) = y_1(x, t) + y_2(x, t) \]
\[ = A' \sin(kx - \omega t + \beta) \]

Using Phasor Diagram!
Wave Interference – quick question!

Speakers A and B emit sound waves with wavelength 1 m, which interfere (fully) constructively at a donkey located far away (say, 200 m). What happens to the sound intensity if speaker A steps back 2.5 m?

If \( l = 1 \) m, then a shift of 2.5 m corresponds to \( 2.5l \), which puts the two waves out of phase, leading to destructive interference. The sound intensity will therefore go to zero.

1) intensity increases  
2) intensity stays the same  
3) intensity goes to zero  
4) impossible to tell
Energy in the Waves

Let’s consider a wave propagating along a string with tension \( T \) and mass per unit length \( \mu \).

\[
\nu^2 = \frac{T}{\mu}
\]

\[
y(x, t) = A \cdot \sin(kx - \omega \cdot t)
\]

\[
y(x, t) = A \cdot \sin[k(x - \nu \cdot t)]
\]

Traveling wave moving in the +x direction.

Is there any moving matter in the +x direction? \textbf{NO!}

Is there any matter moving in the y direction? \textbf{YES!}

Since the particle of the string have mass and moving in the y direction there is kinetic energy associated with this movement!
\[ y(x, t) = A \cdot \sin[k(x - v \cdot t)] \]

\[ dE_{kin} = \frac{1}{2} (dm) v_y^2 = \frac{1}{2} \mu \cdot dx \left( \frac{\partial y}{\partial t} \right)^2 \]

\[ \frac{\partial y}{\partial t} = -Akvcos[k(x - vt)] \]

\[ \left( \frac{\partial y}{\partial t} \right)^2 = \{-Akvcos[k(x - vt)]\}^2 \]

To find out the total kinetic energy in one wave length we need to integrate the \( dE_{kin} \) over a wavelength.
\[ y(x, t) = A \cdot \sin[k(x - v \cdot t)] \]

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\[ E_{kin} = \frac{1}{2} \mu A^2 k^2 v^2 \int_0^\lambda \left[ \cos[k(x - vt)] \right]^2 dx \]

Kinetic energy in one wavelength:

\[ E_{kin} = \frac{1}{2} \mu A^2 \frac{T}{\mu} \frac{4\pi^2 \lambda}{\lambda^2} = \frac{A^2 \pi^2 T}{\lambda} \]
\[ y(x, t) = A \cdot \sin[k(x - v \cdot t)] \]

\[ E_{kin} = \frac{A^2 \pi^2 T}{\lambda} \]

Potential energy?

There is potential energy due to the tension \( T \) that stretches and squeezes the string to give its shape.

Total energy in a traveling wave:

\[ E_{tot} = E_{kin} + U_{pot} = 2 \frac{A^2 \pi^2 T}{\lambda} \]

Energy is always proportional with the squared of the amplitude.
Power on a Transmission Line

\[ \frac{\partial^2 v(x, t)}{\partial t^2} = \frac{1}{LC} \frac{\partial^2 v(x, t)}{\partial x^2} \]

\[ v(x, t) = V \cdot \sin(kx - \omega \cdot t) \]

\[ i(x, t) = I \cdot \sin(kx - \omega \cdot t + \varphi) \]

\[ p = v \cdot i \quad \text{Instantaneous power} \]

\[ P_{av} = \frac{1}{T} \int_0^T p \cdot dt = \frac{1}{2} V I \cos(\varphi) \]

\[ \frac{V}{I} = Z_0 = \sqrt{\frac{L}{C}} \]

\[ L \text{ [Henry/m]} \text{ – inductance per unit length} \]

\[ C \text{ [Farad/m]} \text{ – capacitance per unit length} \]

Characteristic impedance of the transmission line

\[ P_{av} = \frac{1}{2} \frac{V^2}{Z_0} \cos(\varphi) = \frac{1}{2} I^2 Z_0 \cos(\varphi) \]
Electromagnetic Waves

\[ \frac{\partial^2 E(x,t)}{\partial t^2} = \frac{1}{c^2} \frac{\partial^2 E(x,t)}{\partial x^2} \]

\[ \frac{\partial^2 B(y,t)}{\partial t^2} = \frac{1}{c^2} \frac{\partial^2 B(y,t)}{\partial y^2} \]
Optical Fiber Waveguide

If the next index is lower and the incident angle is large enough, the light can be trapped inside.
Transmission Lines

[Image of transmission lines in a field]

[Image of a network cable with colored wires]

[Image of a coaxial cable connector]

[Image of a printed circuit board]

[Image link: "it/e-tlstand.html"]
Transmission Lines

\[
\frac{\partial^2 v(x, t)}{\partial t^2} = \frac{1}{LC} \frac{\partial^2 v(x, t)}{\partial x^2}
\]

\[
\frac{\partial^2 i(x, t)}{\partial t^2} = \frac{1}{LC} \frac{\partial^2 i(x, t)}{\partial x^2}
\]

\[
\frac{V}{I} = Z_0 = \sqrt{\frac{L}{C}}
\]

If the transmission is uniform and infinite, the wave in the +x direction will continue indefinitely and never return in the −x direction.
Transmission line

$Z_0$

$Z_L = 0$

Short circuit

http://www.falstad.com/circuit/e–tlstand.html
Transmission line

Open circuit

\[ Z_0 \quad -x \quad +x \quad Z_L = \infty \]
Transmission line

Matched condition

The matched impedance condition is a unique situation in which all the power of the +x wave is delivered to the load just as if it were an infinite transmission line, with no reflected waves generated in the −x direction.

http://www.falstad.com/circuit/e–tlstand.html