Oscillations and Waves
Lectures 1~3
Interference and Standing Waves

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The aim of the course is to introduce basic topics in **mechanics, fields and waves** for use in subsequent courses on devices, electricity and magnetism and optoelectronics.
Part II (S2): Oscillations and Waves

- Simple Harmonic Motion
- Coupled Oscillators
- Electromagnetic Oscillations
- Travelling Waves and Wave Equation
- Standing Waves and Interference
- Electromagnetic Waves
Course overview

- In class mini exercise
- Tutorial and Revision class
- Final revision week
- Final Examination (50%)
- **Participate! Participate! Participate!**
- Office hour (Wens. 12:00 – 13:00) / Email
Course material

- Online slides
Part II (S2): Oscillations and Waves

- Simple Harmonic Motion
- Coupled Oscillators
- Electromagnetic Oscillations
- Travelling Waves and Wave Equation
- Standing Waves and Interference
- Electromagnetic Waves
Lecture 1

Interference and Standing Wave
By the end of this week’s lectures, you should be able to

- describe standing wave mathematically

Reading: Sections 16.9 -16.12
Principle of Superposition for Waves

The Wave Equation

\[ \frac{d^2 y}{dx^2} = \frac{1}{v^2} \frac{d^2 y}{dt^2} \]

**Question:** Given two waves travelling on the same string

\[ y_1(x, t), y_2(x, t) \]

What would be the resultant wave (net wave)?

\[ y'(x, t) = y_1(x, t) + y_2(x, t) \]

The Principle of Superposition!
Two waves: *same* amplitude, *same* wavelength, *same* direction

\[ y_1(x, t) = y_m \sin(kx - \omega t) \]

\[ y_2(x, t) = y_m \sin(kx - \omega t + \phi) \]

**What is the resultant wave?**

\[ y'(x, t) = y_1(x, t) + y_2(x, t) \]

**Hint:** $\sin \alpha + \sin \beta = 2 \sin \frac{1}{2} (\alpha + \beta) \cos \frac{1}{2} (\alpha - \beta)$
Interference of Waves

Two waves: same amplitude, same wavelength, same direction

\[ y_1(x, t) = y_m \sin(kx - \omega t) \]
\[ y_2(x, t) = y_m \sin(kx - \omega t + \phi) \]

What is the resultant wave?

\[ y'(x, t) = \left[ 2y_m \cos \frac{1}{2} \phi \right] \sin \left( kx - \omega t + \frac{1}{2} \phi \right) \]

Special cases of \( \phi \)?
Interference of Waves

\[ y'(x, t) = [2y_m \cos \frac{1}{2} \phi] \sin(kx - \omega t + \frac{1}{2} \phi) \]
Interference of Waves

Two waves: same wavelength, same direction, different amplitude

\[ y_1(x, t) = y_{m1}\sin(kx - \omega t) \]
\[ y_2(x, t) = y_{m2}\sin(kx - \omega t + \phi) \]

The resultant wave:

\[ y'(x, t) = y_1(x, t) + y_2(x, t) = y'_m\sin(kx - \omega t + \beta) \]

Using Phasor Diagram!
Speakers A and B emit same sound waves with wavelength 1 m, which interfere (fully) constructively at a donkey located far away (say, 200 m). What happens to the sound intensity if speaker A steps back 2.5 m?

If \( l = 1 \) m, then a shift of 2.5 m corresponds to \( 2.5l \), which puts the two waves out of phase, leading to destructive interference. The sound intensity will therefore go to zero.

Follow-up: What if you move back by 4 m?
Standing Waves

Two waves: *same* amplitude, *same* wavelength, *opposite* directions

\[ y_1(x, t) = y_m \sin(kx - \omega t) \]
\[ y_2(x, t) = y_m \sin(kx + \omega t) \]

What is the resultant wave?

\[ y'(x, t) = y_1(x, t) + y_2(x, t) \]

**Hint:** \( \sin \alpha + \sin \beta = 2 \sin \frac{1}{2} (\alpha + \beta) \cos \frac{1}{2} (\alpha - \beta) \)
Standing Waves

Two waves: *same* amplitude, *same* wavelength, *opposite* directions

\[ y_1(x, t) = y_m \sin(kx - \omega t) \]

\[ y_2(x, t) = y_m \sin(kx + \omega t) \]

*The resultant wave:*

\[ y'(x, t) = [2y_m \sin(kx)] \cos(\omega t) \]

*Nodes: \( \sin kx = 0 \Rightarrow x = n \frac{\lambda}{2}, \quad n = 0, 1, 2 \ldots \)*

*Antinodes: \( \sin kx = \pm 1 \Rightarrow x = (n + \frac{1}{2}) \frac{\lambda}{2}, n = 0, 1, 2 \ldots \)*
Standing Waves

\[ y'(x, t) = [2y_m \sin kx] \cos \omega t \]
By the end of this week’s lectures, you should be able to

- describe standing wave mathematically

Reading: Sections 16.9 -16.12
Lecture 2

Standing Wave and Resonance
By the end of this week’s lectures, you should be able to

- explain standing wave and resonance

Reading: Section 16.13 (standing waves and resonance)
Wave Reflection

Experiment
Wave Reflection: Fixed End
Wave Reflection: Free End
Wave Reflection: General Case
Standing Waves and Resonance

Experiment
**Resonance**: the amplitude is peaked when the driving frequency $\omega_d$ nears the natural frequency $\omega$.

**Resonance**: $N$ coupled oscillators, $N$ resonance frequencies $N \to \infty$?
Standing Waves and Resonance

First harmonic

$L = \frac{\lambda}{2}$

Second harmonic

$L = \lambda = \frac{2\lambda}{2}$

Third harmonic

$L = \frac{3\lambda}{2}$
Standing Waves and Resonance

Wavelength

\[ L = n \frac{\lambda}{2} \quad n = 1, 2, 3, \ldots \]

Frequency

\[ f = \frac{v}{\lambda} \quad n = 1, 2, 3, \ldots \]

Harmonic series: \( f_1, f_2, f_3, \ldots \)

Harmonic number: \( n \)
2 minutes’ exercise

A string is clamped at both ends and plucked so it vibrates in a standing mode between two extreme positions a and b. Let upward motion correspond to positive velocities. When the string is in position c, the instantaneous velocity of points on the string:

1) is zero everywhere
2) is positive everywhere
3) is negative everywhere
4) depends on the position along the string

When the string is flat, all points are moving through the equilibrium position and are therefore at their maximum velocity. However, the direction depends on the location of the point. Some points are moving upwards rapidly, and some points are moving downwards rapidly.

Follow-up: positions a and b?
By the end of this week’s lectures, you should be able to

- explain standing wave and resonance

Reading: Section 16.13 (standing waves and resonance)
Lecture 3

Standing Wave Behavior on Transmission Lines (O)
By the end of this week’s lectures, you should be able to

- explain the standing wave behavior on a transmission line
Transmission Lines
KVL:

\[
v(x, t) - L\Delta x \frac{\partial i(x, t)}{\partial t} - v(x + \Delta x, t) = 0
\]

\[
\frac{v(x, t) - v(x + \Delta x, t)}{\Delta x} = L \frac{\partial i(x, t)}{\partial t}
\]

\[
\Delta x \to 0 \Rightarrow -\frac{\partial v(x, t)}{\partial x} = L \frac{\partial i(x, t)}{\partial t}
\]
KCL:

\[ i(x,t) = i(x + \Delta x, t) + C\Delta x \frac{\partial v(x + \Delta x, t)}{\partial t} \]

\[ \frac{i(x, t) - i(x + \Delta x, t)}{\Delta x} = C \frac{\partial i(x, t)}{\partial t} \]

\[ \Delta x \to 0 \Rightarrow -\frac{\partial i(x, t)}{\partial x} = C \frac{\partial v(x, t)}{\partial t} \]
Voltage and Current on Transmission Line

KVL:

\[- \frac{\partial v(x, t)}{\partial x} = L \frac{\partial i(x, t)}{\partial t}\]

KCL:

\[- \frac{\partial i(x, t)}{\partial x} = C \frac{\partial v(x, t)}{\partial t}\]

The above leads to

\[\frac{\partial^2 v(x, t)}{\partial t^2} = \frac{1}{LC} \frac{\partial^2 v(x, t)}{\partial x^2}\]
Voltage and Current Waves

Wave equation

\[
\frac{\partial^2 v(x, t)}{\partial t^2} = \frac{1}{LC} \frac{\partial^2 v(x, t)}{\partial x^2}, \quad \frac{\partial^2 i(x, t)}{\partial t^2} = \frac{1}{LC} \frac{\partial^2 i(x, t)}{\partial x^2}
\]

**Question:** what is the wave speed?
As voltage and current on transmission line are waves, will we observe wave reflection, standing wave behaviours?
Voltage Wave Reflection: Fixed/Free End?

Fixed End
Voltage Wave Reflection: Fixed/Free End?

Free End

\[ V(x, t) \]

\[ Z_L = \infty \]
Voltage Wave Reflection: General Case
Impedance Matching
By the end of this week’s lectures, you should be able to

- explain the standing wave behavior on a transmission line
Oscillations and Waves

Lectures 1~3

Interference and Standing Waves

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