Oscillations and Waves
Lectures 1~3
Electromagnetic Waves

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The aim of the course is to introduce basic topics in mechanics, fields and waves for use in subsequent courses on devices, electricity and magnetism and optoelectronics.
Part II (S2): Oscillations and Waves

- Simple Harmonic Motion
- Coupled Oscillators
- Electromagnetic Oscillations
- Travelling Waves and Wave Equation
- Standing Waves and Interference
- Electromagnetic Waves
Course overview

- In class mini exercise
- Tutorial and Revision class
- Final revision week
- Final Examination (50%)
- Participate! Participate! Participate!
- Office hour (Wens. 12:00 – 13:00)
Course material

- Online slides
- Halliday, D., Resnick, R. and Walker, J. 
  Fundamentals of Physics, John Wiley.
- French, A. P. Vibrations and Waves. New York, 
Part II (S2): Oscillations and Waves

- Simple Harmonic Motion
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- Travelling Waves and Wave Equation
- Standing Waves and Interference
- Electromagnetic Waves
Lecture 1

Electromagnetic Waves and Maxwell’s Equations
By the end of this week’s lectures, you should be able to

- describe electromagnetic waves mathematically

Reading: Sections 33.1-33.4
Fig. 33-1  The electromagnetic spectrum.
Maxwell’s Rainbow: Examples

- Increasing energy $E$
- Increasing frequency $\nu$
- Increasing wavelength $\lambda$

PET scan
Cosmic ray
X-ray
Visible light
Night vision
Dental curing
Remote
Wireless data
Radar
Microwave oven
Ultrasound
Cell phone
AM radio

<table>
<thead>
<tr>
<th>Gamma</th>
<th>X-ray</th>
<th>Ultraviolet</th>
<th>Infrared</th>
<th>Terahertz</th>
<th>Microwave</th>
<th>Broadcast and wireless radio</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-12}$</td>
<td>$10^{-11}$</td>
<td>$10^{-10}$</td>
<td>$10^{-9}$</td>
<td>$10^{-8}$</td>
<td>$10^{-7}$</td>
<td>$10^{-6}$</td>
</tr>
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</table>

Wavelength $\lambda$ (m)
Maxwell’s Rainbow: Visible Spectrum

![Graph showing the relative sensitivity of the average human eye to electromagnetic waves at different wavelengths. This portion of the electromagnetic spectrum to which the eye is sensitive is called visible light.]

**Fig. 33-2** The relative sensitivity of the average human eye to electromagnetic waves at different wavelengths. This portion of the electromagnetic spectrum to which the eye is sensitive is called visible light.
A loop with an AC current produces a changing magnetic field. Two loops have the same area, but one is made of plastic and the other copper. In which of the loops is the induced voltage greater?

Faraday’s law says nothing about the material. The change in flux is the same (and N is the same), so the induced emf is the same.

Follow-up: Current? Without the loop?

1) the plastic loop
2) the copper loop
3) voltage is same in both
### Maxwell’s Equations

<table>
<thead>
<tr>
<th>Name</th>
<th>Equation</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>Gauss’ law for electricity</td>
<td>$\oint \vec{E} \cdot d\vec{A} = q_{\text{enc}}/\varepsilon_0$</td>
<td>Relates net electric flux to net enclosed electric charge</td>
</tr>
<tr>
<td>Gauss’ law for magnetism</td>
<td>$\oint \vec{B} \cdot d\vec{A} = 0$</td>
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<tr>
<td>Faraday’s law</td>
<td>$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$</td>
<td>Relates induced electric field to changing magnetic flux</td>
</tr>
<tr>
<td>Ampere–Maxwell law</td>
<td>$\oint \vec{B} \cdot d\vec{s} = \mu_0\varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{\text{enc}}$</td>
<td>Relates induced magnetic field to changing electric flux and to current</td>
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*Written on the assumption that no dielectric or magnetic materials are present.*
Gauss’s Law for Electricity

Gauss’ law relates the net flux of an electric field through a closed surface (a Gaussian surface) to the net charge $q_{\text{enc}}$ that is enclosed by that surface.

$$\varepsilon_0 \Phi = q_{\text{enc}} \quad \text{(Gauss’ law)}.$$

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{\text{enc}} \quad \text{(Gauss’ law)}.$$

The net charge $q_{\text{enc}}$ is the algebraic sum of all the enclosed positive and negative charges, and it can be positive, negative, or zero.

If $q_{\text{enc}}$ is positive, the net flux is outward; if $q_{\text{enc}}$ is negative, the net flux is inward.

*Fig. 23-6* Two point charges, equal in magnitude but opposite in sign, and the field lines that represent their net electric field. Four Gaussian surfaces are shown in cross section. Surface $S_1$ encloses the positive charge. Surface $S_2$ encloses the negative charge. Surface $S_3$ encloses no charge. Surface $S_4$ encloses both charges and thus no net charge.
Gauss’s Law for Magnetism

\[ \Phi_B = \oint \mathbf{B} \cdot d\mathbf{A} = 0 \quad \text{(Gauss’ law for magnetic fields).} \]

Gauss’ law for magnetic fields holds for structures even if the Gaussian surface does not enclose the entire structure. Gaussian surface II near the bar magnet of Fig. 32-4 encloses no poles, and we can easily conclude that the net magnetic flux through it is zero.

For Gaussian surface I, it may seem to enclose only the north pole of the magnet because it encloses the label N and not the label S. However, a south pole must be associated with the lower boundary of the surface because magnetic field lines enter the surface there. Thus, Gaussian surface I encloses a magnetic dipole, and the net flux through the surface is zero.
Faraday’s Law of Induction

An emf is induced in the loop at the left in Figs. 30-1 and 30-2 when the number of magnetic field lines that pass through the loop is changing.

The magnitude of the emf $\mathcal{E}$ induced in a conducting loop is equal to the rate at which the magnetic flux $\Phi_B$ through that loop changes with time.

$$\oint \vec{E} \cdot d\vec{s} = - \frac{d\Phi_B}{dt}$$ (Faraday’s law).

**Fig. 30-1** An ammeter registers a current in the wire loop when the magnet is moving with respect to the loop.
Here \( i_{\text{enc}} \) is the current encircled by the closed loop.

When there is a current but no change in electric flux (such as with a wire carrying a constant current), the first term on the right side of the second equation is zero, and so it reduces to the first equation, Ampere’s law.
## Maxwell’s Equations

### Table 32-1

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By the end of this week’s lectures, you should be able to

- describe electromagnetic waves mathematically

Reading: Sections 33.1-33.4
Lecture 2

Electromagnetic Waves (A)
By the end of this week’s lectures, you should be able to

- describe electromagnetic waves mathematically

Reading: Sections 33.1-33.4
Fig. 33-3  An arrangement for generating a traveling electromagnetic wave in the shortwave radio region of the spectrum: an LC oscillator produces a sinusoidal current in the antenna, which generates the wave. $P$ is a distant point at which a detector can monitor the wave traveling past it.
Traveling EM Wave
Traveling EM Wave
Fig. 33-3  An arrangement for generating a traveling electromagnetic wave in the shortwave radio region of the spectrum: an $LC$ oscillator produces a sinusoidal current in the antenna, which generates the wave. $P$ is a distant point at which a detector can monitor the wave traveling past it.
The dashed rectangle of dimensions $dx$ and $h$ in Fig. 33-6 is fixed at point $P$ on the $x$ axis and in the $xy$ plane.

As the electromagnetic wave moves rightward past the rectangle, the magnetic flux $B$ through the rectangle changes and—according to Faraday’s law of induction—induced electric fields appear throughout the region of the rectangle. We take $E$ and $E + dE$ to be the induced fields along the two long sides of the rectangle. These induced electric fields are, in fact, the electrical component of the electromagnetic wave.

\[
\oint \vec{E} \cdot d\vec{s} = - \frac{d\Phi_B}{dt} = (E + dE)h - Eh = h \, dE.
\]

\[
\frac{d\Phi_B}{dt} = h \, dx \frac{dB}{dt} \quad \Rightarrow \quad h \, dE = -h \, dx \frac{dB}{dt} \quad \Rightarrow \quad \frac{dE}{dx} = - \frac{dB}{dt}.
\]
Fig. 33-7 The sinusoidal variation of the electric field through this rectangle, located (but not shown) at point $P$ in Fig. 33-5b, $E$ induces magnetic fields along the rectangle. The instant shown is that of Fig. 33-6: is decreasing in magnitude, and the magnitude of the induced magnetic field is greater on the right side of the rectangle than on the left.

\[ \oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}, \]
\[ \oint \vec{B} \cdot d\vec{s} = -(B + dB)h + Bh = -h dB. \]

\[ \Phi_E = (E)(h dx), \]
\[ \frac{d\Phi_E}{dt} = h dx \frac{dE}{dt} \]
\[ -h dB = \mu_0 \varepsilon_0 \left( h dx \frac{dE}{dt} \right) \]
\[ -\frac{\partial B}{\partial x} = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}. \]
ELEC1206: Electrical Materials and Fields
Part II

Traveling EM Wave: Qualitatively

Wave equation!!!

\[ \frac{\partial^2 E}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}, \quad \frac{\partial^2 B}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial t^2} \]
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Reading: Sections 33.1-33.4
Lecture 3

Electromagnetic Waves (B)
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- describe electromagnetic waves mathematically

Reading: Sections 33.1-33.4
Fig. 33-3  An arrangement for generating a traveling electromagnetic wave in the shortwave radio region of the spectrum: an $LC$ oscillator produces a sinusoidal current in the antenna, which generates the wave. $P$ is a distant point at which a detector can monitor the wave traveling past it.
Traveling EM Wave
Traveling EM Wave: Qualitatively

\[ \frac{dE}{dx} = -\frac{dB}{dt} \]
\[ -\frac{\partial B}{\partial x} = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t} \]

Wave equation!!!

\[ \frac{\partial^2 E}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} \]
\[ \frac{\partial^2 B}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial t^2} \]
What is the wave speed?

\[
\frac{\partial^2 E}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}, \quad \frac{\partial^2 B}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial t^2}
\]

- Travelling wave speed

\[
v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}
\]

- In vacuum

\[
\mu_0 = 1.257 \times 10^{-6} \text{ H/m}, \varepsilon_0 = 8.854 \times 10^{-12} \text{ F/m}
\]

- Travelling wave speed

\[
v = c = 2.998 \times 10^8 \text{ m/s}
\]
Traveling EM Wave: Qualitatively

\[ \frac{dE}{dx} = -\frac{dB}{dt} \quad \text{and} \quad \frac{d^2 B}{dx^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}, \]

Wave equation!!!

\[ \frac{\partial^2 E}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}, \quad \frac{\partial^2 B}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial t^2} \]
What is the relationship between $E$ and $B$

- Suppose

$$E = E_m \sin (kx - \omega t + \phi)$$

- What would be $B$?

- We know

$$\frac{\partial E}{\partial x} = - \frac{\partial B}{\partial t}, \quad - \frac{\partial B}{\partial x} = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$$

- Therefore

$$\frac{\partial B}{\partial t} = ?, \quad \frac{\partial B}{\partial x} = ?$$
What is the relationship between $E$ and $B$

- Suppose
  \[ E = E_m \sin (kx - \omega t + \phi) \]

- $B$ satisfies
  \[ \frac{\partial B}{\partial t} = -kE_m \cos(kx - \omega t + \phi) \]
  \[ \frac{\partial B}{\partial x} = \mu_0 \varepsilon_0 \omega E_m \cos(kx - \omega t + \phi) \]

- What would be $B$ then?
  \[ B = \frac{1}{c} E_m \sin(kx - \omega t + \phi) \]
Traveling EM Wave: Qualitatively

\[ \frac{\partial^2 E}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}, \quad \frac{\partial^2 B}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial t^2} \]

\[ B = \frac{1}{c} E_m \sin(kx - \omega t + \phi) = \frac{1}{c} E \]
Transverse or Longitudinal?
Maxwell’s Rainbow

Fig. 33-1  The electromagnetic spectrum.
Further Questions for You to Think About

- EM wave energy transport
- EM wave interference and standing wave
- EM wave in more dimensions
- ...
By the end of this week’s lectures, you should be able to

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