Rotating Machines

Fundamental principles & equations, machine windings
Last Week - Transformers

• Equivalent circuit
• Transformer testing
• Transformer connections
• Transformer construction
• Voltage regulation
• Parallel operation
• Auto-transformer
• Harmonics
• Unbalanced loading
Today

• Why do rotating machines rotate?
• Rotating field principle
• Fundamental torque equation
• Machine windings
  • Stator winding construction
  • Isolation and failure mechanisms
• Coursework information
• Introduction to CAD design for electromagnetics
Rotating Machine Principles

Why do rotating machines rotate, how do they generate electricity.
Some Machine Parts

- Windings – pieces of conductor laid in coils
- Stator – stationary part of the electric machine
- Rotor – rotating part of the electric machine
  - Salient pole – magnetic field generated by coils around distinct poles
    mainly attractive tangential gap forces
  - Non-salient pole – magnetic field generated by coils distributed over slots
    both attractive and repulsive gap forces
- Air gap – distance between stator and rotor, should be as small as possible
- Commutator (DC) – allows changing direction of the current
Most Common Machine Types

Synchronous Machines
- Majority of electrical power generation, very efficient
- Low starting torque, pure synchronous machines cannot start by themselves

Asynchronous Machines
- Cheap and robust, low maintenance, efficient
- Poor speed control, good starting torque

Direct Current Machines
- Easy to control, excellent starting torque
- More maintenance than AC machines
Recap - The Magnetic Field

Four basic principles describe how magnetic fields are used in devices:

• A current-carrying wire produces a magnetic field in the area around it

• A time-changing magnetic field induces a voltage in a coil of wire, if it passes through that coil (This is the basis of the transformer)

• A current-carrying wire in the presence of a magnetic field has a force induced on it (This is the basis of the motor)

• A moving wire in the presence of a magnetic field has a voltage induced in it (This is the basis of the generator)
Assume a static magnetic field with a coil running through it.

Attaching a DC power source to the coil will result in a current flowing round.

Current carrying wire in a magnetic field will result in a force acting on the wire.
Force on a Charged Particle

\[ \vec{F} = q\vec{v} \times \vec{B} = qvB\sin\phi \]

\[ T = \frac{NC}{ms} = \frac{C/A}{Am} = \frac{N}{Am} \]
Force on a Wire (Lorentz Force)

\[ \vec{F} = (q\vec{v} \times \vec{B})nAl \]

\[ I = nqvA \]

\[ \vec{F} = I\vec{l} \times \vec{B} \]

\[ d\vec{F} = Idl \times \vec{B} \]

\( n = \) amount of charge carriers
The Most Basic Rotating Machine

Assume a static magnetic field with a coil running through it.

Attaching a DC power source to the coil will result in a current flowing round.

Current carrying wire in a magnetic field will result in a force acting on the wire.

\[ B = \mu \cdot H \text{ in } T = \frac{N}{m \cdot A} \]

\[ d\vec{F} = ld\vec{l} \times \vec{B} \text{ in } N \]
Torque

\[ F_1 = F_2 = IaB \]
\[ \tau = F_2 b \sin \theta \]
\[ \vec{\mu} = NIA\hat{n} \]
\[ \vec{\tau} = \vec{\mu} \times \vec{B} \]
The resulting **force** causes the coil to rotate for 90° from our starting position.

\[
B = \mu \cdot H \text{ in } T = \frac{N}{m \cdot A}
\]

\[
F_{\text{MAX}} = I \cdot l \cdot B \text{ in } N
\]
The resulting force causes the coil to rotate for $90^\circ$ from our starting position.

In order to rotate further, the force, hence the current must change direction.

This is done by a so-called commutator. This is the basic principle of the DC machine.
What About AC Current?

The force will be maximum when the current hits its peak, have a minimum and will change direction.

\[
B = \mu \cdot H \text{ in } T = \frac{N}{m \cdot A}
\]

\[
d\vec{F} = d\vec{l} \cdot d\vec{l} \times \vec{B} \text{ in } N
\]

\[
F \propto I \cdot B
\]
What About AC Current?

The coil can only rotate if it has a speed *synchronous* with the frequency of the applied current (& voltage).

\[ B = \mu \cdot H \text{ in } T = \frac{N}{m \cdot A} \]

\[ d\vec{F} = d\vec{l} \cdot d\vec{l} \times \vec{B} \text{ in } N \]

\[ F \propto I \cdot B \]
AC Current

The coil can only rotate if it has a rotational speed such that the force is 0 when the coil is vertical.

\[ B = \mu \cdot H \text{ in } T = \frac{N}{m \cdot A} \]

\[ d\vec{F} = d\vec{l} \cdot d\vec{l} \times \vec{B} \text{ in } N \]

\[ F \propto 0 \cdot B \]
AC Current

The coil can only rotate if it has a rotational speed such that the force is 0 when the coil is vertical.

\[ B = \mu \cdot H \text{ in } T = \frac{N}{m \cdot A} \]

\[ d\vec{F} = d\vec{l} \cdot d\vec{l} \times \vec{B} \text{ in } N \]

\[ F \propto I \cdot B \]
Single-Phase AC Generator

A wire rotating in the magnetic field has a voltage induced in it.

With a constant rotational speed $\omega$, a sinusoidal *emf* can be generated.

\[
\begin{align*}
\phi_m &= NBA \cos \theta \\
\theta &= \omega t + \delta
\end{align*}
\]

\[
\Rightarrow \phi_m = NBA \cos(\omega t + \delta)
\]

\[
\varepsilon = -\frac{d\phi_m}{dt} = NBA \omega \sin(\omega t + \delta)
\]
Real Machines

• These examples were very simple and inefficient, real machines have a large number of coils distributed over rotor as well as the stator.

• In synchronous machines the rotor is often carrying the windings generating the magnetic field (excitation winding, with a number of pole pairs), but can also hold permanent magnets

• The stator usually holds the armature windings (main current holding windings), generating a three-phase rotating field.

• DC Machines can have a number of different windings*, more on DC machines later on.

* Armature winding, commutating field winding, series field winding, shunt field winding, compensating winding.
Rotating Field Principle

Three-phase rotating fields.
Types of Fields

Constant

Oscillating

Rotating

Oscillating = 2 x rotating
Rotating Field
Induction Motor

A three-phase AC winding generates a rotating magnetic field.

When a closed conductor loop is located in a changing magnetic field, a emf is induced (Faraday’s law).

As a result a current is flowing in the loop, which creates a magnetic field opposed to the rotating field (Lenz law).

\[ \epsilon = -\frac{\partial \phi}{\partial t} \]
Squirrel Cage

- Laminated rotor bars
- End ring
- Fan (cooling)
Fundamental Torque Equation

Torque between two windings.
Fundamental Torque Equation

A general equation of energy balance:

\[ e_1 = i_1 R_1 + \frac{d}{dt} (L_1 i_1) + \frac{d}{dt} (M i_2) \]
\[ e_2 = i_2 R_1 + \frac{d}{dt} (L_2 i_2) + \frac{d}{dt} (M i_1) \]

\[ T = \frac{1}{2} i_1^2 \frac{dL_1}{d\theta} + \frac{1}{2} i_2^2 \frac{dL_2}{d\theta} + i_1 i_2 \frac{dM}{d\theta} \]

- reluctance torque \((T_r)\)
- excitation torque \((T_e)\)

\( L \) = self-inductance
\( M \) = mutual inductance between the windings
\( \Theta \) = angular position of the rotor
Example: Singly Excited Reluctance Motor

Synchronous motor that starts like an induction machine (no current in secondary coil) and operates at synchronous speed.

Singly-excited: $L_2$ and $M$ terms disappear, hence:

$$T = \frac{1}{2} i_1^2 \frac{dL_1}{d\theta} = \frac{1}{2} i_2^2 \frac{dL_2}{d\theta} + i_1 i_2 \frac{dM}{d\theta}$$

For two pole case, we may assume that the reluctance and thus inductance vary sinusoidal with rotor displacement.

$$L_1 = L_0 + L \cdot \cos(2\theta)$$

, hence:

$$\frac{dL_1}{d\theta} = -2L \cdot \sin(2\theta) \quad i = I_m \cos(\omega t)$$

, then substitute

$$T = -L I_m^2 \cos^2(\omega t) \sin(2\theta) = -\frac{L I_m^2}{2} (\sin(2\theta) + \sin(2\theta) \cos(2\omega t))$$

$$T = -\frac{L I_m^2}{2} \{\sin(2\theta) + \frac{1}{2} \sin(2\theta + 2\omega t) + \frac{1}{2} \sin(2\theta - 2\omega t)\}$$

At synchronous speed:

$$\frac{d\theta}{dt} = \omega \text{ and } \theta = (\omega t - \delta)$$

$$T = -\frac{L I_m^2}{2} \{\sin(2\omega t - 2\delta) + \frac{1}{2} \sin(4\omega t - 2\delta) + \frac{1}{2} \sin(-2\delta)\}$$

As the avg. of time dependent term is zero, we find:

$$T_{avg} = \frac{L I_m^2}{4} \cdot \sin(2\delta)$$
Synchronous Machine

With uniform air gap, \( L_1 \) and \( L_2 \) are constants, hence:

\[
T = i_1 i_2 \frac{dM}{d\theta}
\]

Double excitation:

\[
i_1 = I_1 \quad \text{(DC)}
\]

\[
i_2 = I_2 \cdot \cos(\omega t) \quad \text{with} \quad I_2 = I_{\text{max}}
\]

Now assuming sinusoidal variation of \( M \) with displacement:

\[
M = M_{\text{max}} \cdot \cos(\theta)
\]

\[
\frac{dM}{d\theta} = -M_{\text{max}} \cdot \sin(\theta)
\]

Hence:

\[
T = i_1 i_2 \frac{dM}{d\theta} = I_1 I_2 \cos(\omega t) (-M_{\text{max}} \sin(\theta)) = -\frac{1}{2} I_1 I_2 M_{\text{max}} \{\sin(\theta + \omega t) + \sin(\theta - \omega t)\}
\]

At synchronous speed:

\[
\frac{d\theta}{dt} = \omega \quad \text{and} \quad \theta = (\omega t - \delta)
\]

Hence:

\[
T = -\frac{1}{2} I_1 I_2 M_{\text{max}} \{\sin(2\omega t + \delta) + \sin(-\delta)\}
\]

As the average of time-dependent terms is zero, we find:

\[
T_{\text{avg}} = \frac{1}{2} I_1 I_2 M_{\text{max}} \sin(\delta)
\]

Net average torque
Induction Machine

With uniform air gap and double excitation:

\[ T = i_1i_2 \frac{dM}{d\theta} \]

For sinusoidal AC in both windings:

\[ i_1 = I_1 \cdot \cos(\omega_1 t) \quad i_2 = I_2 \cdot \cos(\omega_2 t) \]

\[ M = M_{\text{max}} \cdot \sin(\theta) \quad \text{with: } \theta = (\omega_3 t) \]

Note that \( \omega_1, \omega_2, \omega_3 \) are all different! Then:

\[ T = i_1i_2 \frac{dM}{d\theta} = (I_1 \cdot \cos(\omega_1 t))(I_2 \cdot \cos(\omega_2 t))(M_{\text{max}} \cdot \cos(\omega_3 t)) \]

Various combinations are possible, condition is sought which leads to a average torque which is not time-dependent.

\[ T = \frac{1}{2} i_1 \frac{dL_1}{d\theta} - \frac{1}{2} i_2 \frac{dL_2}{d\theta} - i_1i_2 \frac{dM}{d\theta} \]

e.g. combining \( \omega_2 \& \omega_3 \) terms:

\[ T = \frac{1}{2} I_1 I_2 M_{\text{max}} \cos(\omega_1 t) \{ \cos(\omega_2 + \omega_3 t) \cos(\omega_2 - \omega_3 t) \} \]

If \( \omega_1 = \omega_2 + \omega_3 \), then:

\[ T = \frac{1}{2} I_1 I_2 M_{\text{max}} \{ \cos^2(\omega_1 t) + \cos(\omega_1 t) \cos(\omega_2 - \omega_3 t) \} \]

\[ T = \frac{1}{2} I_1 I_2 M_{\text{max}} \left\{ \frac{1 + \cos(2\omega_1 t)}{2} + \cos(\omega_1 t) \cos(\omega_2 - \omega_3 t) \right\} \]

And finally:

\[ Net \ average \ torque \]

\[ T = \frac{1}{2} I_1 I_2 M_{\text{max}} \]
Machine Windings

Windings and slots, three-phase windings, winding factors, winding examples.
Common Expressions

• Coil: one or more turns connected in series and in similar magnetic positions.
• Winding: coils (groups) connected in one phase.
• Armature winding: carrying the load current (usually in stator)
• Excitation winding: carrying DC current to generate magnetic field
• Brushes: conductors in contact with rotating part, carrying excitation current
A Coil

Coils for armature windings depend on the size of the machine, generally stator windings (1kW ... 2000 MW) are:

- Random-wound stators (<1kV)
- Form-wound stators (>1kV)
- Form-wound stators using Roebel bars (>50 MVA)
Stator Windings
Construction, Roebel bars, Slots, Distributed Windings.
Stator Winding Construction

magnetic steel, copper conductors and electrical insulation

Cross-section of a three turn stator coil

Multi-turn coil being inserted in a laminated steel stator core
Roebel Bars

The higher the power output or input, the larger (and mechanically stiffer) the coils need to be.

Roebel bars are usually laid in slots, each bar representing half a turn (prevent damage during assembly), which are then connected at their ends.
Slots
Machine Winding Terminology

- Pole pitch ($\tau$): distance between poles in terms of slots.
- Full-pitch winding: if the coil pitch for a winding is equal to the pole pitch.
- Chorded winding: if the pitch of the winding is less than full/pole pitch.
- Single-layer winding: only one coil side place in one slot.
- Double layer winding: two coil sides placed in a single slot.
- Closed winding: closed path around the armature or stator.
- Open winding: no closed path (AC).
More Winding Terminology (AC)

• Balanced winding: number of coils per pole per phase is the same and a whole number.

• Unbalanced winding: number of coils per pole per phase is not a whole number.

• Slot angle ($\alpha_{el}$): 180º/pole pitch (electrical degrees)

• Coil span: distance between two coil sides measured in terms of slots.

$$s = \frac{\text{winding pitch}}{\text{slot angle}}$$
Winding Pitches:

- Back pitch: distance between top and bottom coil sides of a coil measured around the back of the armature.
- Front pitch: distance between two coil sides connected to the same commutator segment.
- Winding pitch: distance between the starts of two consecutive coils measured in terms of coil sides.
- Commutator pitch: distance between two commutator segments.

- Lap winding: successive coils overlap.
- Wave winding: end of one coil is not connected to the beginning of the same coil, but to the beginning of another coil of the same polarity.
- Dummy coils: coils that are placed in slots for mechanical balance, but not connected to any windings.
Winding Factors

General emf equation:
\[ e = -N \frac{d\phi}{dt} \]

Assuming sinusoidal distribution of the flux in the air gap:
\[ \phi = \Phi \cdot \cos \theta \]
where:
\[ \theta = \omega t \]
Hence:
\[ e = \omega N \Phi \sin(\omega t) - N \frac{d\Phi}{dt} \cos(\omega t) \]
= rotational emf + transformer emf

If flux/pole is constant:
\[ e = \omega \cdot N \cdot \Phi \cdot \sin(\omega t) \]
\[ E_{\text{max}} = \omega \cdot N \cdot \Phi = 2\pi f \cdot N \cdot \Phi \]
\[ E_{\text{rms}} = \frac{E_{\text{max}}}{\sqrt{2}} = \frac{2\pi}{\sqrt{2}} f \cdot N \cdot \Phi = 4.44 \cdot f \cdot N \cdot \Phi \]
with N = number of effective turns
Winding Factors II

The phase voltage will be the *phasor sum* of the coil voltages.

The ratio: \[ \frac{\text{phasor sum}}{\text{arithmetic sum}} = k_d \]

is termed **winding distribution factor**:

\[ k_d = \frac{\sin \left( v \cdot q \cdot \frac{\alpha_{el}}{2} \right)}{q \cdot \sin \left( v \cdot \frac{\alpha_{el}}{2} \right)} \]

, with: \( q \) = number of slots per pole per phase

, and: \( \alpha_{el} \) = slot angle (electrical)

If the coils are not full-pitch, then they are *short-chorded*.

The ratio: \[ \frac{\text{phasor sum}}{\text{arithmetic sum}} \]

due to short-chording is termed **winding chording factor**:

\[ k_c = \sin v \frac{s \pi}{\tau \frac{\tau}{2}} \]

, with

\( s \) = coil span

\( \tau \) = pole pitch

\( v = 1 \) ... fundamental

\( v = 3, 5, 7 \ldots \) harmonics
Winding Factors III

The winding factor is then:

\[ k_w = k_d \cdot k_c \leq 1 \]

The *emf* is then:

\[ E = 4.44 \cdot f \cdot N \cdot \Phi \cdot k_w \]

**Examples:**

\[
\begin{array}{|c|cccc|}
\hline
q = & 1 & 2 & 3 & \ldots & \infty \\
\hline
K_{d,1} & 1 & 0.966 & 0.960 & 0.955 \\
K_{d,3} & 1 & 0.707 & 0.667 & 0.636 \\
K_{d,5} & 1 & 0.259 & 0.217 & 0.191 \\
K_{d,7} & 1 & -0.259 & -0.177 & -0.136 \\
\hline
\end{array}
\]

\[
\begin{array}{|c|cccc|}
\hline
S/\tau & 1 & 7/9 & 5/6 & 4/5 \\
\hline
K_{c,1} & 1 & 0.940 & 0.966 & 0.957 \\
K_{c,3} & 1 & 0.500 & 0.707 & 0.588 \\
K_{c,5} & 1 & -0.174 & 0.259 & 0.000 \\
K_{c,7} & 1 & -0.766 & -0.259 & -0.588 \\
\hline
\end{array}
\]
Distributed Windings

Each phase winding occupies a number of consecutive slots:

Example: two slots per pole per phase
3 or 4 slots per pole per phase are also common

\[ q = \text{number of slots per pole per phase} \]
\[ \tau = \text{pole pitch} \]
Examples for Machine Windings

2p = 4 (four poles = two pole pairs)
24 slots
Single layer winding
Example for Machine Windings II

2p = 4 (four poles = two pole pairs)
24 slots
Single layer winding
Double Layer Winding

2p = 4
24 slots
Double-layer winding
Short-Chorded Double Layer Winding

2p = 4
24 slots
Double-layer winding
Rotor Winding
Commonly only excitation windings.
Salient Poles

- Low rpm for hydropower.
- Large number of pole pairs.
Non-Salient Poles

- Two pole rotors for turbogenerators (3000 to 3600 rpm)
Squirrel Cage & Permanent Magnets

Machines don’t necessarily windings on the rotor.
• Less maintenance
• Less flexibility during operation
Electrical Machine Design Process
Design Steps for an Electrical Machine

**Review specifications**
- Required output power (peak power, duty cycle?)
- Voltage/current (current costs a lot more than voltage)
- Efficiency
- Cooling method
- Envelope

**Sizing**
- Machine configuration
- TRV and air gap shear stress
- Electric loading
- Magnetic loading

*Based on empirical values*

**Materials (active electromechanical)**
- Lamination steel
- Conductors
- Magnet material
- Rotor retention
- Sintered cores
- Adhesives
- Potting compounds

**Initial design**
- Poles/slot selection
- Rotor/stator dimensions
- Back EMF (PM)
- Winding design
- Flux distribution
- Turns per coil

**Predicted results**
- Shaft power and torque
- Phase current and voltage
- Efficiency
- PSI in air gap
- Torque vs. speed
- Peak Torque/Power
- Back EMF (PM)
- Losses

Compare Motors
Winding Insulation and Failures

Isolation problems, failure mechanisms.
Common Insulation Materials

- Polyesters
- Epoxy Resins
- Mica
- Mica paper
- Glass fibres
Winding Insulation Failure Processes

- Failures can be caused by an external event (manufacturing error, mis-operation, foreign materials); or winding aging
- External events are not predictable – however winding aging leading to failure may be predicted
- Aging is most likely to affect the electrical insulation
- Winding aging can lead to over 25 separate processes that gradually degrade the insulation
Winding Insulation Failure Processes

• Aging leads to deterioration of the electrical and mechanical strength of the insulation

• The winding fails when the insulation strength decreases below the applied mechanical and electric stresses

• Normally it is a transient electric or mechanical stress caused by voltage or current surge (lightning, power system fault, mis-operation) that determines the actual time of failure

• Since such transients difficult to predict, the prediction of time to failure very difficult

• Instead we can estimate the risk of failure in a time interval
Aging Stresses

• Temperature
• Mechanical, vibration and shock
• Voltage (partial discharge)
• Environmental, oil, water, abrasives, dirt
• May combine and accelerate aging
Long term aging of a stator coil leading to delamination (20 years at 120° C)
8 MW, 6.6 kV Motor Stator Poor Impregnation Causing Partial Discharge (PD)
Reduction in Motor Mass
(N. Glew, IEE Colloquium 99/178, 1999)
Capacitive sensors – 75% level [13-15kV]

75th Percentile of PD results by Manufacturer and Year of Install
13-15kV Air-cooled Machines with 80pF sensors
Insulation Issues

• Stator winding insulation is exposed to various stresses resulting in many different failure processes
• In recent years there is anecdotal information that the insulation life is shorter, possibly due to greater stresses imposed on the insulation
• Inverters are a significant new stress for the insulation